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A MATHEMATICAL MODEL FOR THE MAINTENANCE SCHEDULING PROBLEM WITH DISTURBANCE EFFECT AND SINGLE MAINTENANCE TEAM

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Abstract. Due to the increased complexity, maintenance and resource constraints are generally ignored or only one of them is considered in the machine scheduling literature. However, it is critical to consider these two groups of constraints together, especially in cases where resource needs arise during maintenance. In this study, the unrelated parallel machine scheduling problem is considered under maintenance and resource constraints. In the problem considered, maintenance must be performed within a time window and its duration deteriorates depending on the maintenance time. A mathematical model is proposed to solve this problem. Randomly generated test problems are used to analyze the size to which the proposed model can be solved. With the proposed model, it was possible to obtain solutions for problems with up to 25 jobs.

Keywords: Unrelated parallel machine, scheduling problem, resource constraint, planned maintenance, time distortion.

AMS Subject Classification: 91B10, 91D10.

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1 Introduction

Maintenance activities are important for the production process to proceed without interruption in a way that keeps the efficiency and profitability high for the industry (Dündar et al., 2021). In the manufacturing sector, businesses need to schedule maintenance within the scope of planned maintenance works in order to keep their machines and production equipment in working condition. In planned maintenance, it is determined when the maintenance of each machine or equipment will be carried out. Maintenance time is important in many respects. For example, while very frequent maintenance causes both the machine to stop during the maintenance period and the maintenance costs to increase, infrequent maintenance can lead to malfunctions or a decrease in production efficiency over time and an extension of the maintenance period (breakdowns). Maintenance time and frequency can usually be planned flexibly within a fixed interval or a time window by using past failure data. If a time window in which maintenance can be performed is determined, starting the maintenance at the beginning or end of this time window may affect the maintenance period. This situation is generally referred to as maintenance time-dependent deterioration in the literature.

Maintenance activities often require additional resources such as special devices or a specialized maintenance team. Since it can be very costly to provide these resources, in real life there may be a single maintenance source that will perform the maintenance of all machines. If there is only one source, it is not possible to perform more than one maintenance at the same time. Therefore, care should be taken to ensure that maintenance does not overlap when planning.

In this study, the machine scheduling problem and the maintenance and maintenance resource scheduling problems are considered together. The most commonly discussed machine scheduling problems in the literature are single machine, parallel machine, flow type and shop type scheduling. Scheduling of machines that can do the same job, parallel machine scheduling problem aspectIf parallel machines process a job at different times, they are unrelated. This problem is the most general form of parallel machine scheduling problems.

Although it is important to take maintenance activities and resource constraints into account when scheduling production in modern production systems, the complexity of the problem because it increases, scheduling most studies in the literature assume that machines are always available or ignore resources. As stated by Geurtsen et al (2022), there are few studies that consider both planned maintenance and resources together. Studies that consider maintenance planning have generally considered the case where only one maintenance can be performed on a machine. Unlike the accessible literature, this study is the first to consider the cases of allowing multiple maintenance on a machine, deterioration of maintenance time depending on maintenance time, and having a single maintenance team in the unrelated parallel machine scheduling problem.

In the following section of the study, the literature review is given, in the third section, the problem addressed is defined and the developed mathematical model is presented. Experimental results are discussed in the fourth section, and conclusions and suggestions are discussed in the last section.

2 Literature Review

The first machine scheduling study that considered planned maintenance and resources together was conducted in 2000 (Lee & Chen, 2000). The studies conducted since then are presented in Table 1.

Adding planned maintenance and resource constraints, scheduling your problems complexity Therefore, as can be seen from Table 1, most of the studies that consider these constraints together are in the identical parallel machine environment (P).

The completion time of the last job (C_{enb}) objective function is a manufacturer-oriented objective function since it serves to increase machine efficiency. Therefore, it cannot be ignored by many companies. In addition, this objective function also serves to balance machine loads in parallel machine scheduling problems. Therefore, it is one of the most frequently discussed objective functions in the scheduling literature. Similarly, as seen in Table 1, (C_{enb}) is the most frequently used objective function in studies where planned maintenance and resource constraints are considered together. In this study, the objective function is taken as (C_{enb}) in accordance with the literature.

The scheduling of planned maintenance can be evaluated in two ways; flexible or fixed in a time window. In most of the studies in the relevant literature, maintenance times are assumed to be fixed, that is, the start and end times of maintenance are determined in advance. However, in practice, maintenance start times are usually flexible (Wang & Yu, 2010). This means that the start time of maintenance is not known in advance and must be determined within a given time window. Lee and Chen (2000) and Yoo and Lee (2016) considered the case in an identical parallel machine environment, and Wang and Yu (2010) considered the case in a shop environment where maintenance start times are flexible in a time window. In this study, this situation is addressed for the first time in an unrelated parallel machine scheduling problem.

Table 1: Machine Scheduling Studies Considering Planned Maintenance and Resources Together

Source	***Machine the environment***					***Aim***	***Function***	***Solution the method***
	S	**P**	**R**	**F**	**J**	C_{enb}	*other*	
Touat et al. (2022)	-						-	GLS
Shabtay (2022)	-	-				-	-	OR
Lee and Chen (2000)							-	B&B
Yoo and Lee (2016)		-					-	DP
Belkaid et al. (2014)		-						GA, MIP
						-		
Wong et al. (2012)		-				-		G.A.
Wong et al. (2014)		-				-		G.A.
Wang and Liu (2015)		-				-		G.A.
Fu et al. (2019)		-				-		PSO
Liu and Wang (2016)			-			-	-	PTA
Rebai et al. (2013)			-				-	MIP, GA, H AHP,
Tavana et al. (2015)			-				-	TOPSIS, IP
Li et al. (2022)			-			-		ABC
Lei and He (2022)			-			-		ABC
Aramon Bajestani and Beck				-			-	ITA
(2015)				-			-	NSGA-II, BOPSO GA
Boufellouh and Belkaid (2020)				-		-	-	FBS
Fekri et al. (2023)							-	
Wang and Liu (2010)								
					-	-		
Fu et al. (2017)					-	-		PSO

S: single machine, P: identical parallel machine, R: unrelated parallel machine, F: flow type, J: workshop type, GLS: guided local search, GA: genetic algorithm, B&B: branch-and-bound algorithm, YA: approximation algorithm, PTA: polynomial-time algorithm, DP: dynamic programming, H: heuristic, MIP: mixed integer programming, IP: integer programming, FBS: filtered beam search, PSO: particle swarm algorithm, ABC: artificial bee colony, NSGA-II: non-dominated classification genetic algorithm, BOPSO: two-objective particle swarm algorithm, ITA: integrated two-stage algorithm.

There are five studies in the literature that consider planned maintenance and resource constraints together in the unrelated parallel machine scheduling problem. Liu and Wang (2016) studied the unrelated parallel machine scheduling problem with controllable processing times and deteriorating maintenance times. During the planning horizon, at most one maintenance activity can be planned on each machine. In addition, if the start time of the maintenance activity is delayed, the duration of the maintenance activity to be performed will also increase. They proposed a polynomial-time algorithm to solve the problem. Rebai et al. (2013) considered the unrelated parallel machine scheduling problem with a single maintenance team. Maintenance is performed on each machine once during the planning horizon. The aim is to minimize the preventive maintenance costs by the sum of the weighted completion times of the jobs. They proposed a mathematical model, genetic algorithm, and heuristic algorithm to solve the problem. Tavana et al. (2015) studied the problem of scheduling maintenance on unrelated parallel machines with order-dependent deteriorations in processing times. They proposed a three-stage solution approach to solve the problem. In the first stage, the fuzzy AHP approach was used for the selection of the maintenance team. In the second stage, the TOPSIS approach was used to reduce the multi-objective problem to two objectives. In the third stage, the problem was solved using the proposed two-objective integer linear programming model. Li et al. (2022) considered the unrelated parallel machine scheduling problem in which additional resources and maintenance were taken into account. They proposed an artificial bee colony algorithm to solve the problem. Lei and He (2022) considered the resource-constrained unrelated parallel machine scheduling problem together with the planned maintenance scheduling problem. The objective function is to minimize the completion time of the last job. They proposed an adaptive artificial bee colony algorithm to solve the problem.

When these five studies are evaluated in general, Liu and Wang (2016), Rebai et al. (2013) and Li et al. (2022) assumed that a maximum of one maintenance can be performed on a machine. However, in our study, this assumption was removed and multiple maintenances were allowed in the planning period. Tavana et al. (2015) and Lei and He (2022) did not consider the case of maintenance time deterioration in their studies. From this perspective, the unrelated parallel machine scheduling problem, where multiple maintenances are allowed and maintenance times deteriorate depending on time, was addressed for the first time in the literature in this study.

3 Problem Definition and Mathematical Model

In the problem considered, n jobs must be processed on one of m parallel machines. The processing time of each job on each machine (p_{ji}) may be different. Each machine must be maintained within a time window. In other words, the minimum time (g_i) and maximum time (γ_i) that can pass between two consecutive maintenances are known. If the machine is maintained at the end of the minimum time, the maintenance period is called the normal maintenance period (b_i). As this period increases, the maintenance period also deteriorates and extends within a certain rate (a_i). Therefore, it should be determined when the maintenance will be done within the time window. The time between two consecutive maintenances on a machine is called a period. All jobs require preparation time. While the setup time (r_{ji}) of the first job in each period depends only on the machine, the setup times (s_{hji}) of the second and subsequent jobs depend on both the machine and the row. There is only one maintenance team to perform the maintenance. If maintenance needs to be done on two machines at the same time, the maintenance team can only perform one maintenance. Therefore, maintenance should not overlap as much as possible.

While it is sufficient to determine which sequence of the machine the jobs are assigned to in the classical parallel machine scheduling problem, in the problem under consideration, it must be determined which sequence of the machine the jobs are assigned to in which period. In addition, the need to schedule maintenance on each machine, disruptions due to maintenance time during maintenance periods and overlapping constraints make the problem much more complex than the classical version.

The mathematical model of the problem considered and the definitions of the sets, indices, parameters and decision variables used in this model are given below.

Sets and indices:

$N = \{1, 2, \ldots, n\}$ is the set of jobs. $j, h, k \in N$.

$M = \{1, 2, \ldots, m\}$ is the set of machines. $u, i \in M$.

$L = \{1, 2, \ldots, \Omega\}$ period set. $f, l \in L$.

Parameters:

p_{ji}: Processing time of j^{th} job on i^{th} machine;

r_{ji}: Setup time if job j is the first job on the i^{th} machine;

s_{hji}: Setup time if the transition from job h to job j is made on the i machine;

θ: sufficiently large positive number;

b_i: Normal maintenance period of the i machine;

g_i: The minimum time between two consecutive maintenance operations on the i^{th} machine;

γ_i: The maximum time that can pass between two consecutive maintenance operations on the i^{th} machine;

a_i: Deterioration rate of the maintenance period of the i machine.

Decision variables:

x_{jkli}: 1 if job j is in the k^{th} row in the l^{th} period on the i^{th} machine; 0 otherwise;

π_{li}: If the l^{th} period is to be defined on the i^{th} machine, 1; otherwise, 0;

μ_{li}: If the l^{th} maintenance is to be performed on the i machine, 1; otherwise, 0;

Cj : j completion time of the job;
C_{enb}: completion time of last task $C_{enb} = \ enb\{Cj\}\backslash li$;
w_{flui}: If the fth maintenance on the i machine does not start before the l^{th} maintenance on the i machine is completed, 0; If the l^{th} maintenance on the i machine does not start before the f^{th} maintenance on the i machine is completed, 1.

$$enk\ z \ = \ C_{enb} \tag{1}$$

$$\sum_{k\in N}\sum_{l\in L}\sum_{i\in M} x_{jkli} = 1, \quad \forall j \in N \tag{2}$$

$$\sum_{j\in N} x_{j1li} = \pi_{li}, \qquad \forall l \in L,\ \forall i \in M \tag{3}$$

$$x_{jkli} \le \pi_{li}, \qquad \forall j \in N,\ \forall k \in N,\ \forall l \in L,\ \forall i \in M \tag{4}$$

$$\mu_{li} \le \pi_{li}, \qquad \forall l \in L,\ \forall i \in M \tag{5}$$

$$\mu_{li} \ge \pi_{l+1,i}, \qquad \forall l \in L,\ \forall i \in M \tag{6}$$

$$\sum_{j\in N} x_{j,k+1,li} - \sum_{h\in N} x_{hkli} \le 0, \quad \forall k \in N, \quad k < n, \quad \forall l \in L,\ \forall i \in M \tag{7}$$

$$\pi_{l+1,i} - \pi_{li} \le 0, \qquad \forall l \in L,\ l < \Omega,\ \forall i \in M \tag{8}$$

$$C_j \ge r_{ji} + p_{ji} - \theta\left(1 - x_{j1li}\right), \quad \forall j \in N,\ \forall i \in M \tag{9}$$

$$C_j \ge CB + r_{ij} + p_{ij} - \theta\left(1 - x_{j1li}\right), \quad \forall j \in N,\ \forall l \in L,\ l > 1,\ \forall i \in M \tag{10}$$

$$C_j \ge C_h + s_{hji} + p_{ji} - \theta\left(2 - x_{jkli} - x_{h,k-1,li}\right), \quad \forall h, j, k \in N,\ h \ne j,\ k > 1,\ \forall l \in L,\ \ \forall i \in M \tag{11}$$

$$CP \ge C_j - \theta(1 - \sum_{k\in N} x_{jkli}) \tag{12}$$

$$QP \ge CP - \theta\left(1 - \pi_{li}\right), \quad \forall l \in L,\ l = 1,\ \forall i \in M \tag{13}$$

$$QP \ge CP - CB - \theta\left(1 - \pi_{li}\right), \quad \forall l \in L,\ l > 1,\ \forall i \in M \tag{14}$$

$$QP \ge g_i \pi_{li}, \quad \forall l \in L, \quad \forall i \in M \tag{15}$$

$$QP \le \gamma_i \pi_{li}, \quad \forall l \in L, \quad \forall i \in M \tag{16}$$

$$eB \ge CP, \quad \forall l \in L, \quad \forall i \in M \tag{17}$$

$$CB \ge eB + b_i + \left(QP - g_i\right) a_i - \theta\left(1 - \mu_{li}\right), \quad \forall l \in L,\ \forall i \in M \tag{18}$$

$$yB \geq CB - eB - \theta\left(1 - \mu_{li}\right), \quad \forall l \in L,\ \forall i \in M \tag{19}$$

$$CB \leq eB + \theta\left(2 - \mu_{li} - \mu_{fu}\right) + \theta\omega_{flui}, \quad \forall l \in L,\ \forall f \in L,\ \forall i \in M,\ \forall u \in M,\ u \neq i \tag{20}$$

$$CB \leq eB + \theta\left(2 - \mu_{li} - \mu_{fu}\right) + \theta(1 - \omega_{flui}), \quad \forall l \in L,\ \forall f \in L,\ \forall i \in M,\ \forall u \in M,\ u \neq i \tag{21}$$

$$C_{enb} \geq C_j, \quad \forall j \in N \tag{22}$$

$$x_{jkli} \in \{0, 1\}, \quad \forall j \in N,\ \forall k \in N,\ j \neq k,\ \forall l \in L,\ \forall i \in M \tag{23}$$

$$\pi_{li},\ \mu_{li} \in \{0, 1\}, \quad \forall l \in L,\ \forall i \in M \tag{24}$$

$$w_{flui} \in \{0, 1\}, \quad \forall l \in L,\ \forall f \in L,\ \forall i \in M,\ \forall u \in M,\ u \neq i \tag{25}$$

$$C_j,\ C_{enb} \geq 0, \quad \forall j \in N \tag{26}$$

The objective function (1) is the minimization of the completion time of the last job. Constraint (2) guarantees that the jth job is assigned to a row of a period of a machine. Constraint (3) guarantees that if the lth period is defined on the i^{th} machine, a first job is assigned to this period. Constraint (4) allows that a job can be assigned to this period if the lth period is defined on the ith machine. Constraint (5) ensures that the lth maintenance can be performed on the machine if the lth period is defined on the machine. Constraint (6) guarantees that maintenance is performed before the next period can be defined. Constraint (7) ensures that the jobs on each machine are assigned without skipping a row. Constraint (8) ensures that the periods on the same machine are defined without skipping a row. Constraint (9) is the constraint that calculates the completion times of the jobs assigned to the first row of the first period of each machine. The constraint numbered (10) written for the second and subsequent periods is the calculation of the completion times of the jobs assigned to the first row of the relevant period and the constraint numbered (11) to the second or subsequent row. The constraint numbered (12) is for the calculation of the completion time of the lth period on the ith machine. The constraint numbered (13) is the constraint of calculating the duration of the first period of each machine. The constraint numbered (14) calculates the durations of the second and subsequent periods of the machines. The constraint numbered (15) ensures that the duration of the lth period of the ith machine is at least as long as the shortest period that can pass between two consecutive maintenances on the ith machine, while the constraint numbered (16) ensures that this duration is at most as long as the shortest period that can pass between two consecutive maintenances on the ith machine.

4 Experimental Results

The proposed mathematical model is coded in GAMS 24.0.2. Test problems are solved using CPLEX solver with a time limit of 10800 seconds. All tests are performed on Intel (R) Core (TM) i7-5700HQ. It was built on a computer with a GHz processor and 8 GB memory.

Table 2: Parameter Values for the Example Problem

					i=1										*i=2*					
h/j	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
1	0	83	67	32	44	72	76	76	23	74	0	89	39	75	38	63	76	20	12	39
2	60	0	85	77	30	53	7	50	84	79	41	0	9	97	51	92	56	88	56	32
3	75	84	0	16	43	50	93	14	75	45	40	81	0	58	52	19	19	64	46	60
4	44	75	4	0	70	7	72	36	66	64	91	91	79	0	88	75	69	3	76	95
5	88	9	18	5	0	60	47	40	22	70	40	77	51	27	0	3	92	92	64	92
6	67	79	25	45	29	0	51	65	73	24	15	9	55	79	13	0	4	67	16	31
7	79	41	78	11	51	57	0	62	25	26	25	77	34	98	38	6	0	34	25	65
8	89	47	21	35	99	37	73	0	79	54	78	81	71	41	90	79	22	0	92	11
9	83	15	55	21	62	93	90	14	0	34	31	29	59	55	8	13	49	37	0	52
10	38	10	51	56	89	36	9	59	28	0	54	75	96	35	46	35	69	18	34	0

Table 3: For Example Problem Parameter Values g_i, γ_i, b_i, a_i

I	**gI**	**yI**	**bI**	**aI**
1	16	135	43	1.22
2	61	193	6	1.28

5 Example

In the example, 10 jobs will be scheduled on two parallel machines. The initial job setup times (r_{ji}) and processing times (p_{ji}) of the jobs on a per-machine basis are given in Table 1, the sequence-dependent setup times (s_{hji}) are given in Table 2, and the normal maintenance times (b_i) on a per-machine basis, the minimum (g_i) and maximum (γ_i) times that can elapse between two maintenances, and the failure rates (a_i) are given in Table 5.

The example, where there is only one maintenance team and therefore maintenance overlaps are not desired (a) and the case (b) where there is no resource constraint were solved separately. For both cases, the best solution was achieved in 4632 and 1429 seconds, respectively.

The values are 527.44 and 429.92, respectively.The obtained solutions are given in Table 4 and the Gantt charts corresponding to the relevant solutions are given in Figure 1. As can be seen from Figure 1, when there is only one maintenance team, since maintenance cannot be overlapped, machine 1 (M_1) had to wait for the maintenance on machine 2 (M_2) to be completed before maintenance. Similarly, before the second maintenance on M2, the maintenance on the first machine had to be completed. Therefore, the completion time of the last job increased compared to the case (b) without the maintenance team constraint.

6 Test Results

The test problems were solved with the proposed mathematical model and the results of the 10-task problems are given in Table 5, the results of the 15-task problems are given in Table 6. The results of the 20, 25 and 30-task problems that could be solved are given in Table 7.

As can be seen from Table 7, 6 of the 10-task problems (10-2-1-1, 10-3-1-1, 10-3-1-2, 10-3-1-3, 10-3-3-1, 10-3-3-2) the best solution was found within the time limit. Appropriate solutions were found for the remaining problems. In classical unrelated parallel machine scheduling problems, the fact that only the appropriate solutions of a significant part of the 10-job problems, for which the best solutions could be found in a short time, could be found despite the fact that the time limit in the problem under consideration was not very short, such as 10800 seconds, revealed how much the complexity of the problem increased.

Table 4: Results Obtained for the Sample Problem

		(a)							**(b)**						
1	1	1	1	120	120.00	149.08	318.96	169.88	1	1	120	120.00	120	289.88	169.88
1	2	1	1	97	97.00	97.00	149.08	52.08	1	1	150	150.00	150	269.92	119.92
2	1	1	0	126	444.96	-	-	-	1	0	126	415.88	-	-	-
2	2	1	1	102	251.08	318.96	377.44	58.48	1	0	160	429.92	-	-	-
3	2	1	0	150	527.44	-	-	-	0	0	-	-	-	-	-

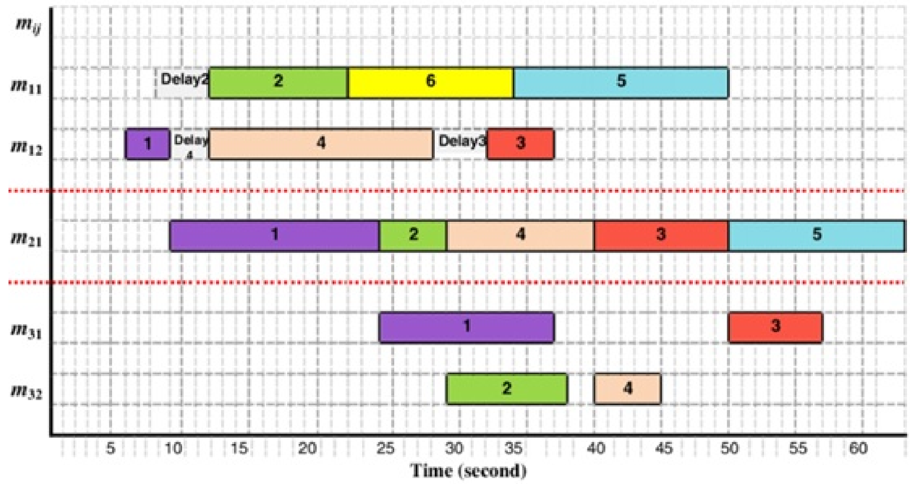

Figure 1: Gantt Chart for Example Problem

Table 5: Results of 10 Worked Problems

m=2			m=3		
problem	**z**	**t**	**problem**	**z**	**t**
10-2-1-1	527.44	7461	10-3-1-1	354.00	7539
10-2-1-2	480.27	10800	10-3-1-2	169.00	2812
10-2-1-3	428.17	10800	10-3-1-3	208.00	6056
10-2-2-1	673.35	10800	10-3-2-1	558.85	10800
10-2-2-2	608.17	10800	10-3-2-2	474.31	10800
10-2-2-3	720.16	10800	10-3-2-3	391.94	10800
10-2-3-1	560.21	10800	10-3-3-1	197.00	4391
10-2-3-2	444.88	10800	10-3-3-2	241.00	6983
10-2-3-3	678.80	10800	10-3-3-3	442.42	10800

Table 6: Results of 15 Worked Problems

m=2			m=3		
problem	**z**	**t**	**problem**	**z**	**t**
15-2-1-1	-	10800	15-3-1-1	-	10800
15-2-1-2	-	10800	15-3-1-2	767.88	10800
15-2-1-3	-	10800	15-3-1-3	-	10800
15-2-2-1	1003.96	10800	15-3-2-1	-	10800
15-2-2-2	1128.31	10800	15-3-2-2	-	10800
15-2-2-3	1106.91	10800	15-3-2-3	441.04	10800
15-2-3-1	1470.68	10800	15-3-3-1	-	10800
15-2-3-2	1373.20	10800	15-3-3-2	-	10800
15-2-3-3	-	10800	15-3-3-3	693.88	10800

Table 7: Results of Problems with 20 and 25 Tasks

problem	z	t
20-2-1-1	1759.86	10800
20-3-1-3	1502.61	10800
25-2-1-3	3418.86	10800

Table 8: Optimal Objective Function Values of 10 Worked Problems

m=2			m=3		
problem	**z**	**t**	**problem**	**z**	**t**
10-2-1-1	527.44	7461	10-3-1-1	354.00	7539
10-2-1-2	480.27	12963	10-3-1-2	169.00	2812
10-2-1-3	428.17	10529	10-3-1-3	208.00	6056
10-2-2-1	673.35	20833	10-3-2-1	522.75	39019
10-2-2-2	608.17	38274	10-3-2-2	474.31	43034
10-2-2-3	708.10	25506	10-3-2-3	391.94	35765
10-2-3-1	560.21	16163	10-3-3-1	197.00	4391
10-2-3-2	444.88	18454	10-3-3-2	241.00	6983
10-2-3-3	678.80	15057	10-3-3-3	437.74	27770

As can be seen from Table 8, none of the 15-task problems could find the best solution within the time limit. Only 8 out of 18 problems could find suitable solutions.

As the problem size increased, the number of problems for which a suitable solution could be obtained decreased rapidly. As can be seen from Table 9, only two of the 20-task problems and only one of the 25-task problems could be solved within the time limit. No solution was found for any of the 30-dimensional problems.

7 Effect of Problem Characteristics on Final Job Completion Time

Since it is necessary to know the best solutions in order to examine the effect of problem characteristics on the completion time of the last job, the best solutions of 10-dimensional problems were obtained by removing the time limit and the obtained results are given in Table 10.

Using the results given in Table 8, variance analysis was performed to determine the effect of the number of machines (m) and the type of deterioration rate (φ) on the completion time of the last job (z). As can be seen from the variance analysis table given in Table 9, both the number of machines (m) and the type of deterioration rate (φ) factors are critical ($p < 0.050$).

As can be seen from the main effects graph given in Figure 1, the completion time of the last job increases when the number of machines is small and the failure rate type is 2. It decreases when the number of machines is large and the failure rate type is 1.

Table 9: Variance Analysis Table

Source	***sd***	***KT***	***KO***	***F***	***p***
m	1	248195	248195	30.37	0,000
φ	2	127406	63703	7.79	0.007
$m * \varphi$	2	3589	1795	0.22	0.806
Mistake	12	98074	8173		
Total	17	477265			

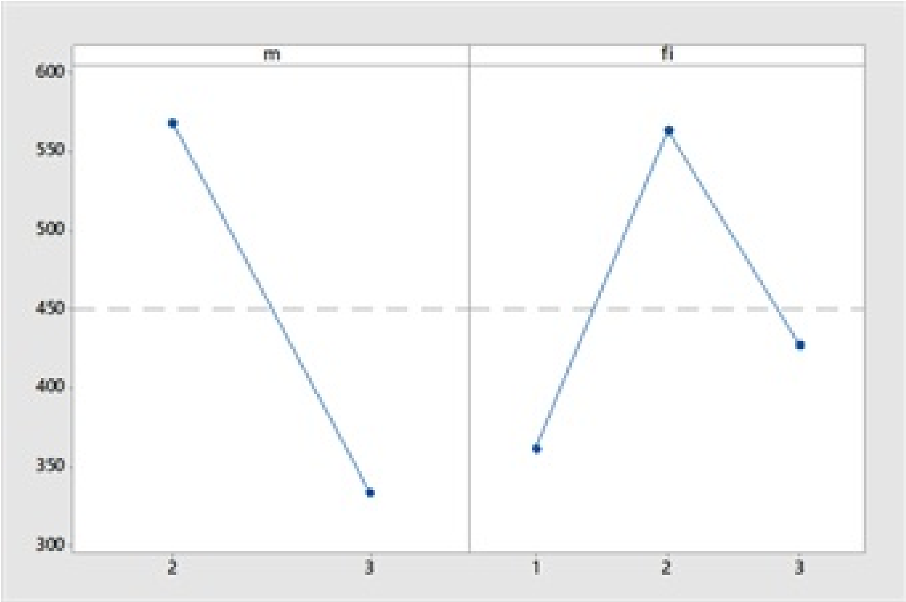

Figure 2: Main Effects Graph

8 Conclusion and Recommendations

In this study, the problem of creating a planned maintenance schedule under a time window constraint is considered. In the problem considered, maintenance times are disrupted depending on the maintenance time. The situation where jobs can be produced on parallel machines in a single-stage production environment is considered. Since the processing times of parallel machines are different, the machines are unrelated. Since the resources used during maintenance are limited, in other words, since there is only one maintenance team that can perform the maintenance, it is not desired to schedule the maintenance simultaneously. A mathematical model is developed for the problem considered. Random test problems of different sizes and features are derived to demonstrate the performance of the developed mathematical model. Problems with up to 25 jobs can be solved within the time limit. In the future, heuristic or metanephric algorithms can be proposed for the problem considered to solve larger-sized problems. In addition, the situation where there is more than one maintenance team can be examined.

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