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A NOVEL COMPUTATIONAL ALGORITHM FOR EFFICIENTLY SOLVING PARTIAL DIFFERENTIAL EQUATIONS

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Abstract. This study introduces a novel approximate analytical method for solving nonlinear partial differential equations, both homogeneous and non-homogeneous. The proposed approach is based on the expansion of solutions using the Taylor series, and a systematic framework is developed for its implementation across a broad class of equations. The convergence behavior of the approximate solution toward the exact solution is analyzed, and the efficiency of the method is demonstrated through numerical examples and graphical illustrations. The results indicate that the proposed method is a promising and effective tool due to its simplicity and high accuracy when compared with traditional techniques.

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1 Introduction

Researchers have been interested for years and decades in the methods and techniques of solving differential equations because of their importance and the need for them in other sciences. Many studies have been devoted to different methods of solving differential and integral equations. For example, Huang et al. (1996) studied the effectiveness of several convergence criteria, when the mixed-form Richards equation was solved using the modified Picard iteration approach, demonstration of the convergence of Adomian's approach in Abbaoui & Cherruault (1994) was novel, when applied to differential equations. Additionally, He (1997) provided new characteristics and formulas for Adomian's polynomials as well as a straightforward computational form, presented a new technique known as the variational iteration approach to solve nonlinear partial differential equations without the need for linearization or minor perturbations, Kreku (2017) suggested a novel local fractional α -integral transform technique to solve the nonlinear fractional gas dynamics equation, which is based on a user-friendly algorithm based on the new homotopy perturbation, Jassim & Vahidi (2021) studied how to solve partial differential equations (PDEs) including local fractional derivative operators (LFDOs) that arise in mathematical physics. The LFDOs serve as the foundation for the reduction differential transform technique (RDTM), which we use to obtain approximations of solutions to these equations.

Many researchers have combined the above methods with integral transformations to avoid the complexities of integrals. In Wang et al. (2014) is combined the Laplace transform with the local fractional Fourier series, Goswami & Alqahtani (2016) combined the Sumudu method with the variational iteration approach and presented new solutions and results based on this approach, in Ahmed et al. (2019) Laplace transform is used with the Adomian approach and modified Adomian approach. There are many studies that combine multiple methods, see for instance, Hussein & Jassim (2023); Jassim et al. (2023); Hussein et al. (2024); Baleanu et al. (2024); Elzaki et al. (2012); Ziane et al. (2018); Al-Essa et al. (2023); Al-Refai & Pal (2019); Jassim et al. (2024); Hussein (2022); Srivastava et al. (2021); Hussein (2024); Renardy & Rogers (2006); Abramowitz et al. (1988); Allami et al. (2018); Kirci et al. (2024); Velieva & Agamalieva (2017).

This manuscript presents a new iterative method for solving partial differential equations. This method is mainly based on Taylor's expansion. In order to introduce the method, we need to mention some concepts and definitions.

An equality involving a function and its derivatives is known as an ordinary differential equation (ODE). The equation of the form is an ordinary differential equation of order n. $F(x, y, y^{'}, ..., y^n) = 0$, where y is a function of $x, y^{'} = \frac{dy}{dx}$is the first derivative with respect to x, and $y^n = \frac{d^n y}{dx^n}$ is the nth derivative with respect to x. A partial differential equation (PDE) is an equation involving functions and their partial derivatives (Renardy & Rogers, 2006).

A Taylor series is a series expansion of a function about a point. A one-dimensional Taylor series is an expansion of a real function $f(x)$ about a point $x = a$ is given by $f(x) = f(a) + f^{'}(a)(x-a) + \frac{f^{''}(a)}{2!}(x-a)^2 + ... + \frac{f^n(a)}{n!}(x-a)^n + ...$

If $a = 0$, the expansion is known as a Maclaurin series (Abramowitz et al., 1988).

2 Analysis of the Method

Consider the following initial value problem in PDE sense

$$\psi^{(n)}(z,t) + L(\psi(z,t)) + N(\psi(z,t)) = g(z,t), n \in N \tag{1}$$

with initial condition $\psi_t^{(i)}(z, 0) = \lambda_i(z), i = 0, 1, 2, ..., n$. Where ψ is an analytical function, $\psi^{(n)} = \frac{\partial^n \psi}{\partial t^n}$, L is a linear operator, N is a nonlinear operator and $g(z,t)$ is a known function. By taking nth integral to both side of Eq. (1),

$$\psi(z,t) = \sum_{k=0}^{n} \lambda_k(z)\frac{t^k}{k!} + I_t^n(g(z,t) - L(\psi(z,t)) - N(\psi(z,t))) \tag{2}$$

Now, we rewrite Eq. (2) by using Maclaurin's expansion with respect to t. Using Maclaurin's extension with respect to t, we formulate Eq. (2).

$$\psi(z,t) = \sum_{i=0}^{\infty} \frac{t^i}{i!} D_t^i (\sum_{k=0}^{n} \lambda_k(z)\frac{t^k}{k!} + I_t^n(g(z,t) - L(\psi(z,t)) - N(\psi(z,t))))_{t=0} \tag{3}$$

where $D_t^i = \frac{d^i}{dt^i}$. Assume that

$$\psi(z,t) = \sum_{i=0}^{\infty} \psi_i, \tag{4}$$

by Substituting Eq. (4) in Eq. (3) gives us the result that

$$\sum_{i=0}^{\infty} \psi_i = \sum_{i=0}^{\infty} \frac{t^i}{i!} D_t^i (\sum_{k=0}^{n} \lambda_k(z)\frac{t^k}{k!} + I_t^n(g(z,t) - L(\sum_{i=0}^{\infty} \psi_i) - N(\sum_{i=0}^{\infty} \psi_i)))_{t=0}. \tag{5}$$

Put $i = i + 1$, Eq. (5) become

$$\psi_0 + \sum_{i=0}^{\infty} \psi_{i+1} = \sum_{i=0}^{\infty} \frac{t^i}{i!} D_t^i (\sum_{k=0}^{n} \lambda_k(z) \frac{t^k}{k!} + I_t^n (g(z,t) - L(\sum_{i=0}^{\infty} \psi_i) - N(\sum_{i=0}^{\infty} \psi_i)))_{t=0}. \tag{6}$$

By comparing the two sides of Eq. (6)

$$\begin{aligned}
&\psi_0 = \psi(z,0) = \lambda_0(z)\\
&\psi_1 = t\left(\lambda_1(z) + D_t^{1-n}\left(g(z,t) - L(\psi_0) - N(\psi_0)\right)_{t=0}\right)\\
&\psi_2 = \frac{t^2}{2!}\left(\lambda_2(z) + D_t^{2-n}\left(g(z,t) - L(\psi_0+\psi_1) - N(\psi_0+\psi_1)\right)_{t=0}\right)\\
&\psi_3 = \frac{t^3}{3!}\left(\lambda_3(z) + D_t^{3-n}\left(g(z,t) - L(\psi_0+\psi_1+\psi_2) - N(\psi_0+\psi_1+\psi_2)\right)_{t=0}\right)\\
&\quad\vdots\\
&\psi_{i+1} = \frac{t^{i+1}}{(i+1)!} D_t^{i+1}\left(\sum_{k=0}^{n} \lambda_k(z)\frac{t^k}{k!} + I_t^n\left(g(z,t) - L\left(\sum_{j=0}^{i}\psi_j\right) - N\left(\sum_{j=0}^{i}\psi_j\right)\right)\right)_{t=0}
\end{aligned} \tag{7}$$

Consequently, the approximate solution is
$\psi(z,t) = \psi_0 + \psi_1 + \psi_2 + ... = \sum_{i=0}^{\infty} \psi_i$.

3 Convergence of the Method

Theorem 1. *The new decomposition method used the solution of Eq. (1) is equivalent to determining the following sequence;*

$$\begin{aligned}
&\varsigma_\eta = \psi_1 + \psi_2 + \psi_3 + ... + \psi_\eta\\
&\varsigma_0 = 0
\end{aligned} \tag{8}$$

By using the iterative scheme:

$$\varsigma_{\eta+1} = \sum_{i=1}^{\eta+1} \frac{t^i}{i!} D_t^i \left(\sum_{k=0}^{n} \lambda_k(z)\frac{t^k}{k!} + I_t^n\left(g(z,t) - L\left(\sum_{j=0}^{i-1}\psi_j\right) - N\left(\sum_{j=0}^{i-1}\psi_j\right)\right)\right)_{t=0} \tag{9}$$

Proof. For $\eta = 0$, from Eq. (9), we have; $\varsigma_1 = t(\lambda_1(z) + D_t^{1-n}(g(z,t) - L(\psi_0) - N(\psi_0))_{t=0})$, then $\lambda_1 = t(\lambda_1(z) + D_t^{1-n}(g(z,t) - L(\psi_0) - N(\psi_0))_{t=0})$. For $\eta = 1$ and by Eq. (9), $\varsigma_2 = \sum_{i=1}^{2} \frac{t^i}{i!}\left(\sum_{k=0}^{n} \lambda_k(z) D_t^i \frac{t^k}{k!} + D_t^{i-n}\left(g(z,t) - L\left(\sum_{j=0}^{i}\psi_j\right) - N\left(\sum_{j=0}^{i}\psi_j\right)\right)\right)_{t=0}$,

$= t(\lambda_1(z) + D_t^{1-n}(g(z,t) - L(\psi_0) - N(\psi_0))_{t=0}) + \frac{t^2}{2!}(\lambda_2(z) + D_t^{2-n}(g(z,t) - L(\psi_0 + \psi_1) - N(\psi_0 + \psi_1))_{t=0})$,
$= \psi_1 + \frac{t^2}{2!}(\lambda_2(z) + D_t^{2-n}(g(z,t) - L(\psi_0 + \psi_1) - N(\psi_0 + \psi_1))_{t=0})$. According to $\varsigma_2 = \psi_1 + \psi_2$, we get; $\psi_2 = \frac{t^2}{2!}(\lambda_2(z) + D_t^{2-n}(g(z,t) - L(\psi_0 + \psi_1) - N(\psi_0 + \psi_1))_{t=0})$. The proof of this theorem will be established through the method of strong induction. Let's assume that $\psi_{m+1} = \frac{t^{m+1}}{(m+1)!} D_t^{m+1}\left(\sum_{k=0}^{n} \lambda_k(z)\frac{t^k}{k!} + I_t^n\left(g(z,t) - L\left(\sum_{j=0}^{m}\psi_j\right) - N\left(\sum_{j=0}^{m}\psi_j\right)\right)\right)_{t=0}$, where $m = 1, 2, 3, \ldots, \eta - 1$, so
$\varsigma_{\eta+1} = \sum_{i=1}^{\eta+1} \frac{t^i}{i!} D_t^i\left(\sum_{k=0}^{n} \lambda_k(z)\frac{t^k}{k!} + I_t^n\left(g(z,t) - L\left(\sum_{j=0}^{i-1}\psi_j\right) - N\left(\sum_{j=0}^{i-1}\psi_j\right)\right)\right)_{t=0}$,

$= t(\lambda_1(z)+D_t^{1-n}(g(z,t)-L(\psi_0)-N(\psi_0))_{t=0})+\frac{t^2}{2!}(\lambda_2(z)+D_t^{2-n}(g(z,t)-L(\psi_0+\psi_1)-N(\psi_0+\psi_1))_{t=0}) + ... + \frac{t^{\eta+1}}{(\eta+1)!}D_t^{\eta+1}\left(\sum_{k=0}^{n}\lambda_k(z)\frac{t^k}{k!} + I_t^n\left(g(z,t) - L\left(\sum_{j=0}^{\eta}\psi_j\right) - N\left(\sum_{j=0}^{\eta}\psi_j\right)\right)\right)_{t=0}$

$= \psi_1+...+\psi_\eta+\frac{t^{\eta+1}}{(\eta+1)!}D_t^{\eta+1}\left(\sum_{k=0}^{n}\lambda_k(z)\frac{t^k}{k!} + I_t^n\left(g(z,t) - L\left(\sum_{j=0}^{\eta}\psi_j\right) - N\left(\sum_{j=0}^{\eta}\psi_j\right)\right)\right)_{t=0}$.

Then, from Eq. (8), it can be driven;

$$\varsigma_{\eta+1} = \frac{t^{\eta+1}}{(\eta+1)!}D_t^{\eta+1}\left(\sum_{k=0}^{n}\lambda_k(z)\frac{t^k}{k!} + I_t^n\left(g(z,t) - L\left(\sum_{j=0}^{\eta}\psi_j\right) - N\left(\sum_{j=0}^{\eta}\psi_j\right)\right)\right)_{t=0} \quad \square$$

Theorem 2. *Let B be a Banach space.*

I. $\sum_{i=0}^{\infty}\psi_i$ in Eq. (7) convergence to $\varsigma \in B$, if

$$\exists(0 \leq \epsilon \leq 1), such that(\forall \eta \in N \Rightarrow ||\psi_\eta|| \leq \epsilon||\psi_{\eta+1}||) \tag{10}$$

II. $\varsigma = \sum_{\eta=1}^{\infty}\psi_\eta$,satisfies in

$$\varsigma = \sum_{i=1}^{\infty}\frac{t^i}{i!}D_t^i\left(\sum_{k=0}^{n}\lambda_k(z)\frac{t^k}{k!} + I_t^n\left(g(z,t) - L\left(\sum_{i=0}^{\infty}\psi_i\right) - N\left(\sum_{i=0}^{\infty}\psi_i\right)\right)\right)_{t=0} \tag{11}$$

Proof. I. From Eq. (10), we have

$$||\varsigma_{\eta+1}-\varsigma_\eta|| = ||\sum_{i=0}^{\eta+1}\psi_i-\sum_{i=0}^{\eta}\psi_i|| = ||\psi_{\eta+1}|| \leq \epsilon||\psi_\eta|| \leq \epsilon^2||\psi_{\eta-1}|| \leq \epsilon^3||\psi_{\eta-2}|| \leq ... \leq \epsilon^{\eta+1}||\psi_0||. \tag{12}$$

for all $\eta, m \in N, \eta \geq m$,

$$\begin{aligned} ||\varsigma_\eta - \varsigma_m|| &= ||(\varsigma_\eta - \varsigma_{\eta-1}) + (\varsigma_{\eta-1} - \varsigma_{\eta-2}) + ... + (\varsigma_{m+1} - \varsigma_m)|| \\ &\leq ||\varsigma_\eta - \varsigma_{\eta-1}|| + ||\varsigma_{\eta-1} - \varsigma_{\eta-2}|| + ... + ||\varsigma_{m+1} - \varsigma_m|| \\ &\leq \epsilon^\eta||\psi_0|| \leq \epsilon^{\eta-1}||\psi_0|| \leq \epsilon^{\eta-2}||\psi_0|| \leq ... \leq \epsilon^{m+1}||\psi_0|| \\ &\leq \epsilon^{m+1}||\psi_0||(\epsilon^{\eta-m-1} + \epsilon^{\eta-m-2} + ... + 1) \\ &= \frac{1-\epsilon^{\eta-m}}{1-\epsilon}\epsilon^{m+1}||\psi_0|| \end{aligned} \tag{13}$$

Then,
$\lim_{\eta,m\to\infty}||\varsigma_\eta - \varsigma_m|| = 0$.
Thus, $\lim_{\eta\to\infty}\varsigma_\eta = \sum_{\eta=1}^{\infty}\psi_\eta = \varsigma$.

II. From Eq. (9),
$\lim_{\eta\to\infty}\varsigma_{\eta+1} =$
$= \lim_{\eta\to\infty}\sum_{i=1}^{\eta+1}\frac{t^i}{i!}D_t^i\left(\sum_{k=0}^{n}\lambda_k(z)\frac{t^k}{k!} + I_t^n\left(g(z,t) - L\left(\sum_{j=0}^{i-1}\psi_j\right) - N\left(\sum_{j=0}^{i-1}\psi_j\right)\right)\right)_{t=0}$
$= \sum_{i=1}^{\infty}\frac{t^i}{i!}D_t^i\left(\sum_{k=0}^{n}\lambda_k(z)\frac{t^k}{k!} + I_t^n\left(g(z,t) - L\left(\sum_{j=0}^{i-1}\psi_j\right) - N\left(\sum_{j=0}^{i-1}\psi_j\right)\right)\right)_{t=0}$
Thus,
$\varsigma = \sum_{i=1}^{\infty}\frac{t^i}{i!}D_t^i\left(\sum_{k=0}^{n}\lambda_k(z)\frac{t^k}{k!} + I_t^n\left(g(z,t) - L\left(\sum_{i=0}^{\infty}\psi_i\right) - N\left(\sum_{i=0}^{\infty}\psi_i\right)\right)\right)_{t=0} = \sum_{i=1}^{\infty}\psi_i$.
$\square$

Theorem 3. *The following Eq. (11)*
$\varsigma = \sum_{i=1}^{\infty}\frac{t^i}{i!}D_t^i\left(\sum_{k=0}^{n}\lambda_k(z)\frac{t^k}{k!} + I_t^n\left(g(z,t) - L\left(\sum_{j=0}^{\infty}\psi_j\right) - N\left(\sum_{j=0}^{\infty}\psi_j\right)\right)\right)_{t=0}$
is equivalent to Eq. (1);
$\psi^{(n)}(z,t) + L(\psi(z,t)) + N(\psi(z,t)) = g(z,t)$, $\psi^{(i)}(z,0) = \lambda_i(z), i = 0,1,2,...,n$

Proof. We rewrite Eq. (11) as fallows;
$\lambda_0(z)+\varsigma=\lambda_0(z)+\sum_{i=1}^{\infty}\frac{t^i}{i!}D_t^i\left(\sum_{k=0}^{n}\lambda_k(z)\frac{t^k}{k!}+I_t^n\left(g(z,t)-L\left(\sum_{i=0}^{\infty}\psi_i\right)-N\left(\sum_{i=0}^{\infty}\psi_i\right)\right)\right)_{t=0}$
$=\sum_{i=1}^{\infty}\frac{t^i}{i!}D_t^i\left(\sum_{k=0}^{n}\lambda_k(z)\frac{t^k}{k!}+I_t^n\left(g(z,t)-L\left(\sum_{i=0}^{\infty}\psi_i\right)-N\left(\sum_{i=0}^{\infty}\psi_i\right)\right)\right)_{t=0}$
By considering $\psi=\varsigma+\lambda_0(z)=\sum_{i=1}^{\infty}\psi_i+\psi_0=\sum_{i=1}^{\infty}\psi_i$, we get

$$\sum_{i=1}^{\infty}\psi_i=\sum_{i=1}^{\infty}\frac{t^i}{i!}D_t^i\left(\sum_{k=0}^{n}\lambda_k(z)\frac{t^k}{k!}+I_t^n\left(g(z,t)-L\left(\sum_{i=0}^{\infty}\psi_i\right)-N\left(\sum_{i=0}^{\infty}\psi_i\right)\right)\right)_{t=0} \tag{14}$$

It appears evident that the expression on the right-hand side of Eq. (14) corresponds to Maclaurin's expansion. Therefore, Eq. (14) can be represented in the following manner.

$$\sum_{i=0}^{\infty}\psi_i=\sum_{k=0}^{n}\lambda_k(z)\frac{t^k}{k!}+I_t^n\left(g(z,t)-L\left(\sum_{i=0}^{\infty}\psi_i\right)-N\left(\sum_{i=0}^{\infty}\psi_i\right)\right) \tag{15}$$

by using Eq. (4), Eq. (15) becomes as follows

$$\psi=\sum_{k=0}^{n}\lambda_k(z)\frac{t^k}{k!}+I_t^n\left(g(z,t)-L\left(\psi\right)-N\left(\psi\right)\right) \tag{16}$$

By taking the nth order derivative from both sides of Eq. (16) $\psi^{(n)}(z,t)+L(\psi(z,t)+N(\psi(z,t))=g(z,t)$ Then solution of Eq. (11) is the same as solution of Eq. (1). ☐

4 Illustrative examples

Example 1. *Consider PDE*

$$\psi_t(z,t)+\psi_z(z,t)=\psi(z,t), \tag{17}$$

with $\psi(z,0)=1+e^z$. *Taking the integration of both sides*

$$\psi=1+e^z+I_t(\psi-\psi_z), \tag{18}$$

From algorithm of the HJ-method,

$$\psi=\sum_{i=0}^{\infty}\frac{t^i}{i!}D_t^i(1+e^z+I_t(\psi-\psi_z))_{t=0}, \tag{19}$$

Suppose that

$$\psi=\sum_{i=0}^{\infty}\psi_i, \tag{20}$$

by Substituting Eq. (20) in Eq. (19)

$$\sum_{i=0}^{\infty}\psi_i=\sum_{i=0}^{\infty}\frac{t^i}{i!}D_t^i(1+e^z+I_t(\sum_{j=0}^{i-1}\psi_j-(\sum_{j=0}^{i-1}\psi_j)_z))_{t=0}. \tag{21}$$

By comparing both side of Eq. (20)

$$\psi_0 = 1 + e^z,$$
$$\psi_1 = tD_t(1 + e^z + I_t(1))_{t=0} = t,$$
$$\psi_2 = \frac{t^2}{2!}D_t^2(1 + e^z + I_t(1 + t))_{t=0} = \frac{t^2}{2!},$$
$$\psi_3 = \frac{t^3}{3!}D_t^3(1 + e^z + I_t(1 + t + \frac{t^2}{2!}))_{t=0} = \frac{t^3}{3!},$$
$$\vdots$$
$$\psi_{i+1} = \frac{t^{i+1}}{(i+1)!}D_t^{i+1}(1 + e^z + I_t(\sum_{j=0}^{i} \frac{t^j}{j!}))_{t=0} = \frac{t^{i+1}}{(i+1)!},$$

Thus, the approximate solution of Eq. (17) is
$\psi_a(z,t) = e^z + 1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + ... = \sum_{i=0}^{\infty} \frac{t^i}{i!}$ *The exact solution of Eq. (17) is* $\psi_e(z,t) = e^z + e^t$

Table 1: Values of approximate and exact solution of Eq. (17) at different values of t and z.

t	z	ψ_α	ψ_e	$\lvert\psi_\alpha - \psi_e\rvert$
0.001000000000000	0.003000000000000	2.004005004670085	2.004005004670085	0.000000000000000
0.053578947368421	0.055473684210526	2.112081480925514	2.112081480966312	0.000000000040798
0.106157894736842	0.107947368421053	2.225986551794815	2.225986554026723	0.000000002231907
0.158736842105263	0.160421052631579	2.346034537928947	2.346034562154561	0.000000024225614
0.211315789473684	0.212894736842105	2.472556662690240	2.472556796047861	0.000000133357621
0.263894736842105	0.265368421052632	2.605901909010865	2.605902413050081	0.000000504039216
0.316473684210526	0.317842105263158	2.746437901031779	2.746439400706755	0.000001499674976
0.369052631578947	0.370315789473684	2.894551811859982	2.894555592287273	0.000003780427290
0.421631578947368	0.422789473684211	3.050651298854067	3.050659737074285	0.000008438220218
0.474210526315789	0.475263157894737	3.215165467924214	3.215182628374389	0.000017160450175
0.526789473684211	0.527736842105263	3.388545868413004	3.388578292363052	0.000032423950048
0.579368421052632	0.580210526315790	3.571267520207964	3.571325241044565	0.000057720836601
0.631947368421053	0.632684210526316	3.763829974825907	3.763927792784833	0.000097817958926
0.684526315789474	0.685157894736842	3.966758412303069	3.966917464061209	0.000159051758140
0.737105263157895	0.737631578947368	4.180604775824020	4.180854436270165	0.000249660446145
0.789684210526316	0.790105263157895	4.405948946126740	4.406329101640718	0.000380155513978
0.842263157894737	0.842578947368421	4.643399957831185	4.643963692519829	0.000563734688645
0.894842105263158	0.895052631578947	4.893597259954609	4.894413998526066	0.000816738571457
0.947421052631579	0.947526315789474	5.157212022999124	5.158371176310347	0.001159153311224
1.000000000000000	1.000000000000000	5.434948495125711	5.436563656918091	0.001615161792380

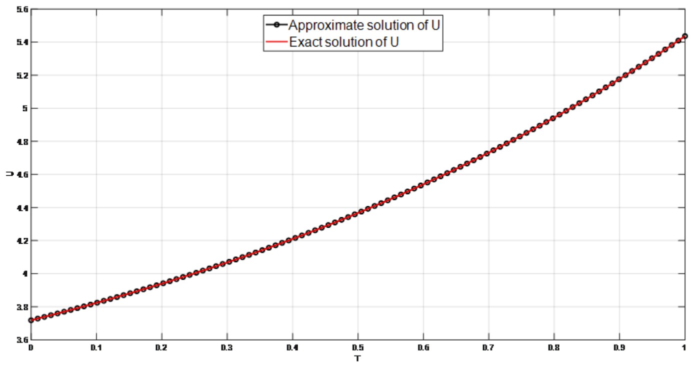

Figure 1: The plot of approximate and exact solution of Eq. (17) at $z = 1$.

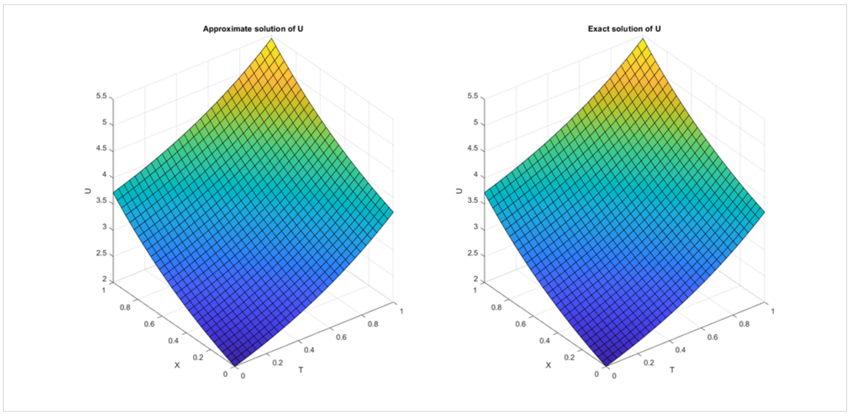

Figure 2: The surface of approximate and exact solution of Eq. (17)

Example 2. *Assume the nonlinear homogeneous differential equation (Burger's equation)*

$$\psi_t(z,t) - \psi_{zz}(z,t) + \psi(z,t)\psi_t(z,t) = 0, \tag{22}$$

with $\psi(z,t) = z$. *By using the above algorithm of the new method,*

$$\sum_{i=0}^{\infty} \psi_i = \sum_{i=0}^{\infty} \frac{t^i}{i!} D_t^i (z + I_t((\sum_{j=0}^{i-1} \psi_j)_{zz} - (\sum_{j=0}^{i-1} \psi_j)(\sum_{j=0}^{i-1} \psi_j)_z))_{t=0}. \tag{23}$$

By comparing both side of Eq. (23)

$$\begin{aligned} \psi_0 &= z, \\ \psi_1 &= tD_t(z + I_t(-z))_{t=0} = -zt, \\ \psi_2 &= \frac{t^2}{2!} D_t^2 (z + I_t(-z(1-t)^2)_{t=0} = -zt^2, \\ \psi_3 &= \frac{t^3}{3!} D_t^3 (-z(1-t+t^2)^2)_{t=0} = -zt^3, \\ &. \\ &. \\ &. \\ \psi_{i+1} &= \frac{t^{i+1}}{(i+1)!} D_t^{i+1} (z + I_t((\sum_{j=0}^{i} \psi_j)_{zz} - (\sum_{j=0}^{i} \psi_j)(\sum_{j=0}^{i} \psi_j)_z))_{t=0} = z(-1)^{i+1}, \end{aligned}$$

the approximate solution is
$\psi_a(z,t) = z(1 - t + t^2 + ...) = z \sum_{i=0}^{\infty} (-1)^i$.
The exact solution is
$\psi_e(z,t) = \frac{z}{1+t}$

Table 2: The values of approximate and exact solution of Eq. (22) at different values of t and z

t	z	ψ_α	ψ_e	$\lvert\psi_\alpha - \psi_e\rvert$
0.001000000000000	0.003000000000000	0.000997009027081	0.000997008973081	0.000000000054000
0.014105263157895	0.016000000000000	0.013883248609088	0.013883133029424	0.000000115579664
0.027210526315789	0.029000000000000	0.026444988546397	0.026443660170835	0.000001328375562
0.040315789473684	0.042000000000000	0.038696761004033	0.038690776846146	0.000005984157887
0.053421052631579	0.055000000000000	0.050653897070685	0.050636068845099	0.000017828225586
0.066526315789474	0.068000000000000	0.062332581151542	0.062290557855312	0.000042023296229
0.079631578947368	0.081000000000000	0.073749908864562	0.073664735381469	0.000085173483092
0.092736842105263	0.094000000000000	0.084923948440191	0.084768594246127	0.000155354194064
0.105842105263158	0.107000000000000	0.095873805624522	0.095611657870965	0.000262147753557
0.118947368421053	0.120000000000000	0.106619692085895	0.106203007518797	0.000416684567098
0.132052631578947	0.133000000000000	0.117182997324941	0.116551307660148	0.000631689664794
0.145157894736842	0.146000000000000	0.127586364088072	0.126664829613300	0.000921534474771
0.158263157894737	0.159000000000000	0.137853767284403	0.136551473593388	0.001302293691015
0.171368421052632	0.172000000000000	0.148010596406132	0.146218789294054	0.001791807112078
0.184473684210526	0.185000000000000	0.158083741452349	0.155673995114368	0.002409746337981
0.197578947368421	0.198000000000000	0.168101682356295	0.164923996133907	0.003177686222388
0.210684210526316	0.211000000000000	0.178094581916060	0.173975400930071	0.004119180985989
0.223789473684211	0.224000000000000	0.188094382228725	0.182834537323701	0.005259844905024
0.236894736842105	0.237000000000000	0.198134904627951	0.191507467131855	0.006627437496096
0.250000000000000	0.250000000000000	0.208251953125000	0.200000000000000	0.008251953125000

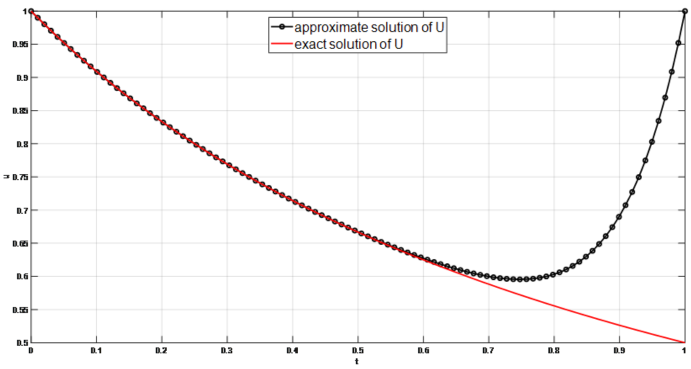

Figure 3: The graphs of approximate and exact solution of Eq. (22) at $z = 1$

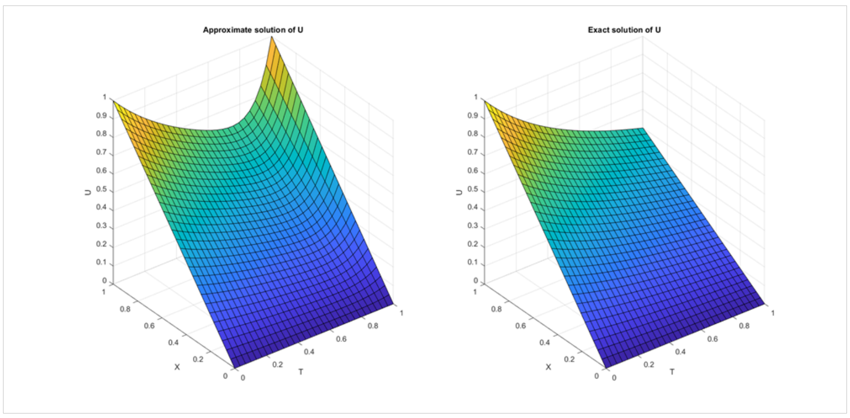

Figure 4: The surface of approximate and exact solution of Eq. (22)

Example 3. *Suppose that the nonhomogeneous linear differential equation of the second order*

$$\psi_{tt}(z,t) - \psi_{zz}(z,t) = sin(z), \tag{24}$$

with $\psi(z,0) = \psi_t(z,0) = sin(z)$, *By algorithm of the method,*

$$\sum_{i=0}^{\infty} \psi_i = \sum_{i=0}^{\infty} \frac{t^i}{i!} D_t^i (sin(z) + tsin(z) + \frac{t^2}{2!} sin(z) + I_t^2 (\sum_{i=0}^{i-1} \psi_i)_{zz})_{t=0} \tag{25}$$

By comparing

$$\psi_0 = sin(z),$$
$$\psi_1 = tD_t(sin(z) + tsin(z) + \frac{t^2}{2!} sin(z) + I_t^2(-sin(z)))_{t=0} = tsin(z),$$
$$\psi_2 = \frac{t^2}{2!} D_t^2 (sin(z) + tsin(z) + \frac{t^2}{2!} sin(z) + I_t^2(-sin(z)(1+t)))_{t=0} = 0,$$
$$\psi_3 = \frac{t^3}{3!} D_t^3 (sin(z) + tsin(z) + \frac{t^2}{2!} sin(z) + I_t^2(-sin(z)(1+t+\frac{t^2}{2!})))_{t=0} = -\frac{t^3}{3!} sin(z),$$
$$.$$
$$.$$
$$.$$

the approximate solution is $\psi_a(z,t) = sin(z) + tsin(z) + tsin(z) - \frac{t^3}{3!} sin(z) + ...$
$= sin(z) + sin(z)(t - \frac{t^3}{3!} + ...))$
The exact solution is
$\psi_e(z,t) = sin(z) + sin(z)sin(t)$

Table 3: The values of approximate and exact solution of Eq. (24) at different values of t and z

t	z	ψ_α	ψ_e	$\lvert\psi_\alpha - \psi_e\rvert$
0.001000000000000	0.003000000000000	0.001002999828333	0.001000999833000	0.000001999995333
0.053578947368421	0.055473684210526	0.056522592463780	0.056421273840757	0.000101318623023
0.106157894736842	0.107947368421053	0.117374369004807	0.117185844325565	0.000188524679242
0.158736842105263	0.160421052631579	0.183320357033870	0.183057514623186	0.000262842410684
0.211315789473684	0.212894736842105	0.254063996651703	0.253740238772060	0.000323757879643
0.263894736842105	0.265368421052632	0.329252221807828	0.328881199037451	0.000371022770377
0.316473684210526	0.317842105263158	0.408478176825278	0.408073520653267	0.000404656172011
0.369052631578947	0.370315789473684	0.491284542933939	0.490859597059305	0.000424945874635
0.421631578947368	0.422789473684211	0.577167443195098	0.576734992130253	0.000432451064845
0.474210526315789	0.475263157894737	0.665580888040600	0.665152879472557	0.000428008568043
0.526789473684211	0.527736842105263	0.755941717818726	0.755528972877267	0.000412744941459
0.579368421052632	0.580210526315790	0.847634993288379	0.847246896527651	0.000388096760727
0.631947368421053	0.632684210526316	0.940019779983442	0.939663938630318	0.000355841353123
0.684526315789474	0.685157894736842	1.032435267827049	1.032117127822683	0.000318140004365
0.737105263157895	0.737631578947368	1.124207163354277	1.123929568055645	0.000277595298632
0.789684210526316	0.790105263157895	1.214654288440767	1.214416964698939	0.000237323741828
0.842263157894737	0.842578947368421	1.303095316569103	1.302894272400683	0.000201044168420
0.894842105263158	0.895052631578947	1.388855575425145	1.388682393776886	0.000173181648260
0.947421052631579	0.947526315789474	1.471273843028700	1.471114857327572	0.000158985701128
1.000000000000000	1.000000000000000	1.549709063687876	1.549544403081468	0.000164660606408

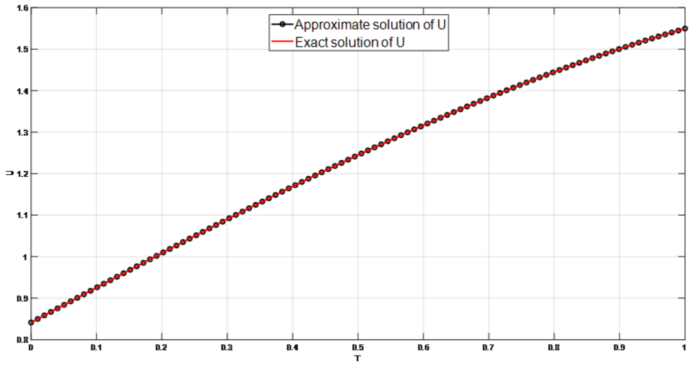

Figure 5: The graphs of approximate and exact solution of Eq. (24) at $z = 1$

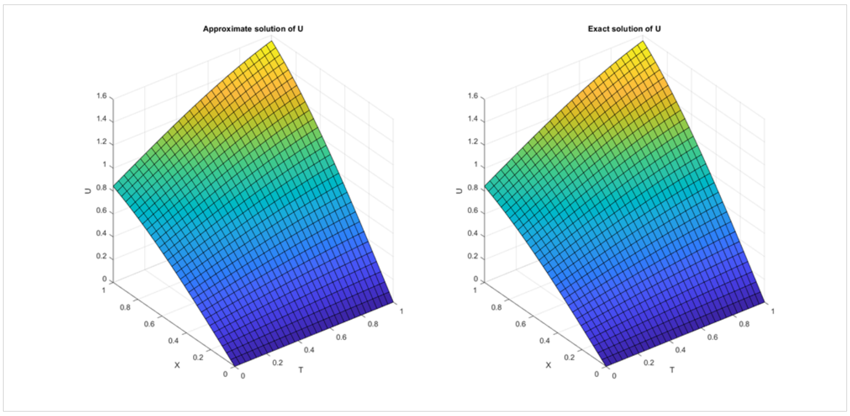

Figure 6: The surface of approximate and exact solution of Eq. (24)

Example 4. *Consider the linear homogeneous PDE of the second order*

$$\psi_{tt}(z,t) - \psi^2_{zz}(z,t) + \psi^2(z,t) = 0 \tag{26}$$

with $\psi(z,0) = 0, \psi_t(z,0) = cos(z)$.
By algorithm of the new method, Eq. (26) become as follows

$$\sum_{i=0}^{\infty} \psi_i = \sum_{i=0}^{\infty} \frac{t^i}{i!} D_t^i (tcos(z) + I_t^2((\sum_{i=0}^{i-1} \psi_i)^2_{zz}) - (\sum_{i=0}^{i-1} \psi_i)^2)_{t=0} \tag{27}$$

By comparing both side of Eq. (27)

$$\begin{aligned} \psi_0 &= 0, \\ \psi_1 &= tD_t(tcos(z) + I_t^2(0))_{t=0} = tcos(z), \\ \psi_2 &= \frac{t^2}{2!} D_t^2(tcos(z) + I_t^2(0))_{t=0} = 0, \\ \psi_3 &= \frac{t^3}{3!} D_t^3(tcos(z) + I_t^2(0))_{t=0} = 0, \\ &. \\ &. \\ &. \end{aligned}$$

$$\psi_{i+1} = \frac{t^{i+1}}{(i+1)!} D_t^{i+1}(tcos(z) + I_t^2((\sum_{j=0}^{i} \psi_j)^2_{zz}) - (\sum_{j=0}^{i} \psi_j)^2))_{t=0} = 0$$

the approximate solution is $\psi_a(z,t) = tcos(z)$
The exact solution is
$\psi_e(z,t) = tcos(z)$

Table 4: The values of approximate and exact solution of Eq. (26) at different values of t and z

t	z	ψ_α	ψ_e	$\lvert\psi_\alpha - \psi_e\rvert$
0.001000000000000	0.003000000000000	0.002999998500000	0.002999998500000	0
0 0.053578947368421	0.055473684210526	0.055394079004324	0.055394079004324	0
0 0.106157894736842	0.107947368421053	0.107339683075742	0.107339683075742	0
0.158736842105263	0.160421052631579	0.158404197412905	0.158404197412905	0
0.211315789473684	0.212894736842105	0.208159059229457	0.208159059229457	0
0.263894736842105	0.265368421052632	0.256181735179365	0.256181735179365	0
0.316473684210526	0.317842105263158	0.302057676957296	0.302057676957296	0
0.369052631578947	0.370315789473684	0.345382245991638	0.345382245991638	0
0.421631578947368	0.422789473684211	0.385762599757636	0.385762599757636	0
0.474210526315789	0.475263157894737	0.422819532374563	0.422819532374563	0
0.526789473684211	0.527736842105263	0.456189262313430	0.456189262313430	0
0.579368421052632	0.580210526315790	0.485525160229939	0.485525160229939	0
0.631947368421053	0.632684210526316	0.510499410150547	0.510499410150547	0
0.684526315789474	0.685157894736842	0.530804597476857	0.530804597476857	0
0.737105263157895	0.737631578947368	0.546155217534388	0.546155217534388	0
0.789684210526316	0.790105263157895	0.556289098675158	0.556289098675158	0
0.842263157894737	0.842578947368421	0.560968734248427	0.560968734248427	0
0.894842105263158	0.895052631578947	0.559982518079516	0.559982518079516	0
0.947421052631579	0.947526315789474	0.553145878441533	0.553145878441533	0
1.000000000000000	1.000000000000000	0.540302305868140	0.540302305868140	0

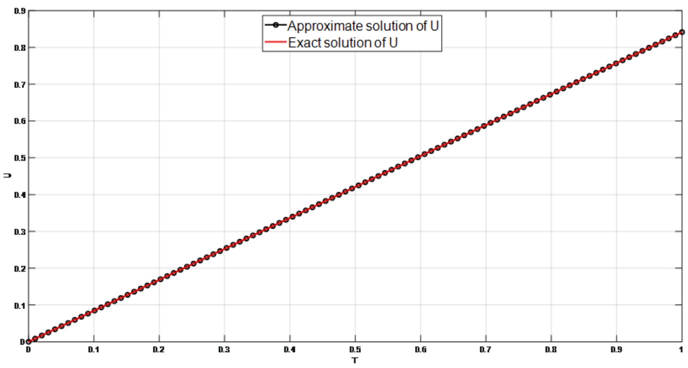

Figure 7: The graphs of approximate and exact solution of Eq. (26) at $z = 1$

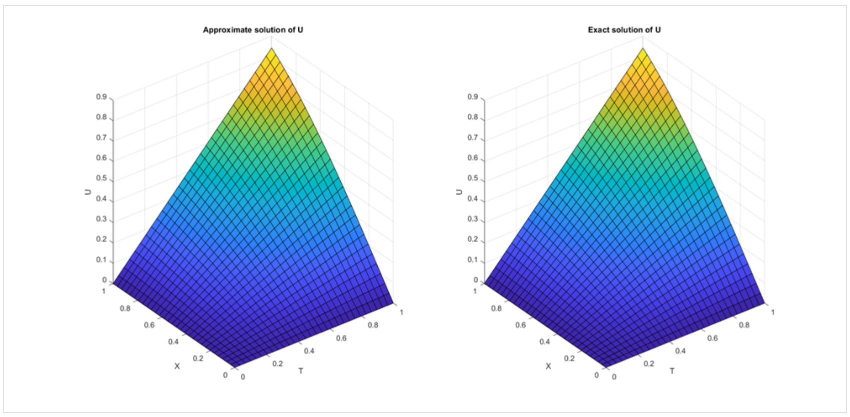

Figure 8: The surface of approximate and exact solution of Eq. (26)

5 Conclusion

This study introduces a new method for solving partial differential equations. the new computational method for solving linear, nonlinear, homogeneous, and nonhomogeneous partial differential equations represents a significant advancement in the field of mathematical analysis. This method has proven to be effective in obtaining solutions for a wide range of PDEs with different orders. The results have demonstrated that the solutions generated by this method converge towards the exact solutions of the equations. Furthermore, this method holds the potential for further development to tackle ordinary differential equations, integral equations, and fractional differential equations. Overall, the new computational method offers a promising approach for solving complex PDEs, opening doors for enhanced problem-solving capabilities and expanding the possibilities for mathematical modeling and scientific exploration.

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