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DOUBLE ACCEPTANCE SAMPLING PLANS FOR GAMMA LINDLEY DISTRIBUTION BASED ON TRUNCATED LIFETIME TESTS

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Abstract. Assuming that the lifetime of an item follows the Gamma Lindley (G-L) distribution, in this paper a double acceptance sampling plan (DASP) is suggested based on truncated life duration testing. The system with zero and two failures is typically addressed in this paper. The minimum sample sizes of the first and second samples' are determined in order to claim that the real mean life is no longer the given life at the selected consumer confidence level. Study is also done on the operation characteristic values and the minimum ratios of mean life to fixed life. Examples are provided to demonstrate the findings in this study. It is found that for a given , the minimum sample sizes decrease with decreasing confidence levels. Researchers in the industry are advised to utilize the specified DASP for the G-L distribution.

Keywords: Acceptance sampling plan, double acceptance sampling plan, consumer's risk, time truncated life tests, Gamma Lindley distribution, operating characteristic function.

AMS Subject Classification: 62P30, 62D05.

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1 Introduction

Acceptance sampling plans (ASP) are essential tools in the area of quality control and assurance because they offer an organized method to evaluate the quality of a manufacturing batch or lot. Such tools are intended to achieve a balance between the price of examining each item in a batch and the risk of delivering faulty goods to customers. They are often utilized in many different businesses to make judgments on whether to accept or reject a batch based on a sample of products. ASP includes three crucial components: sample selection, inspection criteria and decision rules. Based on the inspection results, decision criteria are utilized to determine whether to accept or reject the entire batch. These rules consider factors such as sample size, the number of defects found, and the acceptable level of risk.

In DASP, which differs from the single ASP (SASP), the inspection procedure is divided into two stages: the first stage involves sampling a batch of items and determining whether it meets

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an established acceptance criterion, typically based on the number of defects. The second stage further refines the decision by examining a second sample from the accepted items from the first stage. The main goal of this two-stage inspection procedure is to balance lowering the risk of passing defective items with decreasing inspection costs.

In their research, a number of researchers took the double acceptance sampling plan into account for various lifetime distributions. For example, single, double and multiple sampling plans for the hypergeometric distribution is proposed by Sathakathulla & Murthy (2012). The DASP based on truncated life tests in generalized exponential distribution is suggested by Rao (2016). Malathi & Muthulakshmi (2015) proposed DASP in terms of truncated life tests for the Marshall-Olkin extended exponential distribution. An economic optimal double sampling design with zero acceptance numbers is utilized by Fallahnezhad et al. (2015). Aslam et al. (2016) considered DASP based on truncated life test for some popular statistical distributions. The DASP for time-truncated life tests based on half normal distribution is considered by Al-Omari et al. (2016b). Al-Omari et al. (2016a) proposed DASP for time truncated life tests based on transmuted new Weibull-Pareto distribution. Gui & Lu (2018) suggested DASP based on the Burr type X distribution under truncated life tests. (2018) introduced DASP for percentiles based on the inverse Rayleigh distribution. Al-Omari & Zamanzade (2017) offered DASP for time truncated life tests based on transmuted generalized inverse Weibull distribution. Hamurkaroğlu et al. (2020) proposed single and DASP based on the time truncated life tests for the compound Weibull-exponential distribution. Saha et al. (2021) suggested single and DASP for truncated life tests based on transmuted Rayleigh distribution. Tripath et al. (2021) introduced DASP and GASP for truncated life test based on inverse log-logistic distribution. Sridhar Babu et al. (2021) suggested a DASP for exponentiated Fréchet distribution when the shape parameters are known Al-Omari & Alomani (2022) introduced DASP based on truncated life tests for two-parameter Xgamma distribution. Jayalakshmi & Neena Krishna (2022) considered the problem of designing DASP for truncated life tests based on percentiles using Kumaraswamy exponentiated Rayleigh distribution. Sivakumar (2023) suggested DASP for odd generalized exponential log-logistic distribution based on truncated life tests.

Assuming that a product's lifespan follows the G-L distribution, the goal of this research is to develop DASP for short life testing. Al-Omari & Ismail (2024) suggested group ASP for the G-L distribution and to the authors' knowledge, this study is the first to incorporate the DASP for the G-L distribution.

The rest of the paper is structured as follows. Section 2 describes the G-L distribution with some main properties. The suggested DASP for the G-L distribution is offered in Section 3. The operating characteristic function (OC) and its fundamental properties are presented in Section 4. The minimum mean ratios to the specified life time and the corresponding producer's risk values are investigated in Section 5 with some illustrated examples. The usage of tables and examples are discussed in Section 6. A summary of the important issues appears in the last section.

2 The G-L distribution

Due to the importance of probability distribution functions in modeling various real data, such as in economics, engineering, industry, biology, and medicine, many new distributions are suggested to address this need. One such model is the gamma Lindley distribution introduced by Zeghdoudi & Nedjar (2016) and further studied by Nedjar & Zeghdoudi (2016) to explore additional properties of the distribution. Shafq et al. (2022) added further details to the Gamma Lindley distribution and introduced a key condition on the distribution's probability density function (pdf), which is $\lambda > \frac{\alpha}{\alpha+1}$. The pdf of the G-L distribution is defined as: 1

$$l(y;\alpha,\lambda)=\frac{\alpha^2}{\lambda(1+\alpha)}\left[(\lambda+\lambda\alpha-\alpha)y+1\right]e^{-\alpha y}, y,\alpha>0,\lambda>\frac{\alpha}{\alpha+1}, \tag{1}$$

with a cumulative distribution function (cdf) given by

$$L(y;\alpha,\lambda)=1-\left(\frac{(\alpha\,\lambda+\lambda-\alpha)(\alpha y+1)+\alpha}{\lambda(\alpha+1)}\right)e^{-\alpha y}. \tag{2}$$

The r-th moment and the mean of the G-L distribution, respectively, are given by $E\left(Y^r\right)=\frac{\Gamma(r+1)}{\alpha^r(\lambda\alpha+\lambda)}\left[\lambda(r+1)(\alpha+1)-r\alpha\right]$, $r=1,2,3,...$ and $E(Y)=\frac{2\lambda(\alpha+1)-\alpha}{\alpha(\lambda\alpha+\lambda)}$.

Figure 1 shows some pdf plots for the G-L distribution for several parameter values. The plot indicates that the G-L distribution has different shapes and shows its flexibility in modeling real data sets. Also, the G-L distribution is positively skewed with different levels of skewness based on the parameter values.

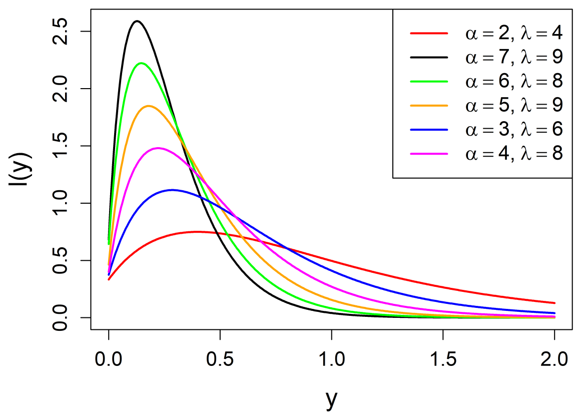

Figure 1: The pdf plots of the G-L for some parameters

The G-L distribution is a unimodal with mode given by $M=\frac{\lambda\alpha+\lambda-2\alpha}{\theta(\lambda+\lambda\alpha-\alpha)}$, $\lambda\in\left[\frac{2\alpha}{\alpha+1},\infty\right]$, and zero otherwise. The moments estimate of the G-L distribution parameters are found to be $\hat{\alpha}=\frac{1}{s^2+m^2}\left(2m+\sqrt{2\left(m^2-s^2\right)}\right)$ and $\hat{\lambda}=\frac{\hat{\alpha}}{(1+\hat{\alpha})(2-\hat{\alpha}m)}$, where m is the first-moment m and s^2 is the variance. The reliability and hazard rate functions of the G-L distribution, respectively, are given by

$$R(y;\alpha,\lambda)=1-L(y;\alpha,\lambda)=\frac{(\alpha\,\lambda+\lambda-\alpha)(\alpha y+1)+\alpha}{\lambda(\alpha+1)}e^{-\alpha y},$$

and

$$H(y;\alpha,\lambda)=\frac{l(y;\alpha,\lambda)}{R(y;\alpha,\lambda)}=\frac{\alpha^2\left[(\lambda+\lambda\alpha-\alpha)y+1\right]}{(\alpha\,\lambda+\lambda-\alpha)(\alpha y+1)+\alpha}.$$

Figure 2 includes the hazard and reliability function plots of the G-L distribution for various selections of the parameters. The hazard function of the distribution is increasing while the reliability function is decreasing for all choices of the parameter values.

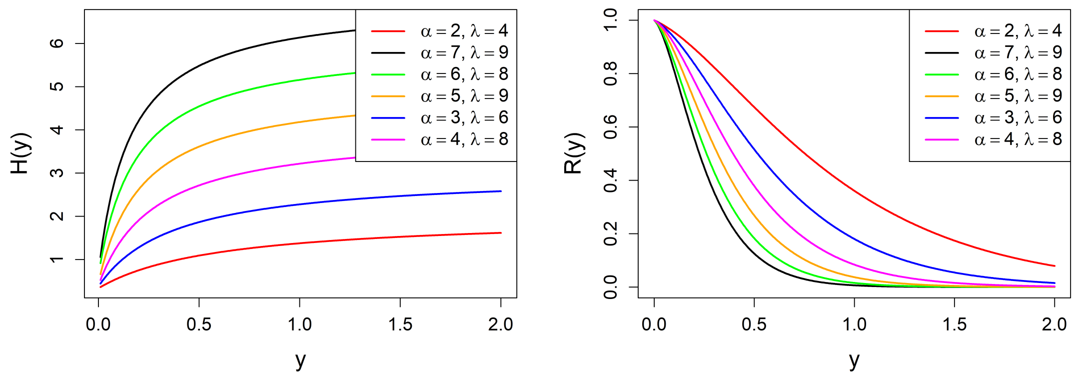

Figure 2: The hazard and reliability plots of the G-L distribution for some parameters

3 Double acceptance sampling plan

In this section, the suggested DASP is illustrated assuming that the lifetime of the test follows the Gamma-Lindley (G-L) distribution. In general, waiting until a product fails completely before assessing it may take some time. To save money and effort, it's standard procedure to end a life test at a certain time t_0.

Among the objectives of these tests are to define a certain mean life, μ, with a probability of α or more (the consumer's degree of confidence), and to establish a confidence limit on the mean of the lifetime. The lot is considered good if the real mean life does not fall below a preset value μ_0, which satisfies the hypothesis $H_0 : \mu \geq \mu_0$ against $H_1 : \mu < \mu_0$. The lot will only be accepted if the observed number of failures falls under a predetermined acceptance number c. The test may be terminated at time t_0, and the lot may be rejected if there are more failures than this number. Determining the minimal sample size n necessary to ensure a specific mean lifespan is the topic under discussion. The probability that an inadequate lot will be accepted and a good lot will be rejected for a specific acceptance sampling plan are known as the consumer's risk and the producer's risk, respectively.

The DASP in terms of truncated lifetime can be described as follows:

1. Draw a first random sample of size n_1 and put them on test within time t_0. If there are c_1 or fewer failures, accept the lot. If $c_2 + 1$ failures are detected, end the test and reject the lot; i.e., $c_1 < c_2$.

2. If the number of failures by time t_1 is between $c_1 + 1$ and c_2, then select a second sample of size n_2, and test the selected items during another time t_0. If at most c_2 failures are detected from the two samples, i.e., $n_1 + n_2$, accept the lot. Otherwise, the process is terminated, and the lot is rejected.

With reference to the above steps, the DASP involves parameters (n_1, c_1, n_2, c_2), and the DASP can be denoted by $\left(n_1, c_1, n_2, c_2, \frac{t_0}{\mu_0}\right)$. Assuming that the lot is large enough to apply the binomial probability distribution, based on the DASP, the probability of accepting the lot is given by

$$p_{DASP} = \sum_{i=0}^{c_1} \binom{n_1}{i} p^i (1-p)^{n_1-i} + \sum_{i=c_1+1}^{c_2} \binom{n_1}{i} p^i (1-p)^{n_1-i} \times \left(\sum_{k=0}^{c_2-i} \binom{n_2}{k} p^k (1-p)^{n_2-k} \right) \tag{3}$$

where $p = L\,(t;\mu) = L\left(\frac{t}{\mu_0}.\frac{\mu_0}{\mu}\right)$ is given by the cdf in Equation 2. For $c_1 = 0$ and $c_2 = 1$ in the DASP we get the so called zero and one failure technique. Particularly, in this study we consider the zero and two failure scheme, that is $c_1 = 0$ and $c_2 = 2$. The probability of acceptance of the lot respectively denoted by p_1 and p_2 for the sampling plans $\left(n_1, c_1, \frac{t}{\mu_0}\right)$ and $\left(n_2, c_2, \frac{t}{\mu_0}\right)$ are given by

$$p_1 = \sum_{i=0}^{c_1=0} \binom{n_1}{i} p^i (1-p)^{n_1-i}, \tag{4}$$

and

$$p_2 = \sum_{k=0}^{c_2=2} \binom{n_2}{k} p^k (1-p)^{n_2-k}. \tag{5}$$

Hence, the probability of acceptance can be obtained by using (4) and (5) as P(no failure arises in the 1st sample) + P(1 failure arises in the 1st sample and 0 or 1 failure arises in 2nd sample) + P(2 failures arises in the 1st sample and 0 or 1 failure arises in the 2nd sample).

4 Operating characteristic function

The operating characteristic function (OC) is a key tool in the acceptance sampling plans since it gives the probability of accepting a lot. The OC determines the effectiveness of a statistical hypothesis test in order to decide whether to accept or reject a lot. The sampling strategy is preferred by the researchers if its OC rises quickly to one.

Table 1: OC values of the sampling plan $(n_1, c_1 = 0, t/\mu_0)$ for a given P^* under the G-L distribution with $\alpha = 2, \lambda = 24$

P^*	t/μ_0	n_1	μ/μ_0					
			2	4	6	8	10	12
0.75	0.628	4	0.553269	0.830351	0.910181	0.943249	0.960189	0.970103
	0.942	2	0.565044	0.832232	0.911236	0.944305	0.961287	0.971210
	1.257	2	0.407576	0.743532	0.859861	0.911123	0.938056	0.953974
	1.571	1	0.532361	0.807979	0.896226	0.934518	0.954539	0.966346
	2.356	1	0.319200	0.666435	0.808038	0.875347	0.912208	0.934541
	3.141	1	0.181183	0.532522	0.713607	0.808068	0.862383	0.896278
	3.927	1	0.099144	0.415784	0.620217	0.737273	0.808014	0.853529
	4.712	1	0.052938	0.319200	0.532468	0.666435	0.751549	0.808038
0.90	0.628	6	0.411533	0.756646	0.868344	0.916092	0.940882	0.955491
	0.942	3	0.424741	0.759219	0.869854	0.917632	0.942496	0.957127
	1.257	2	0.407576	0.743532	0.859861	0.911123	0.938056	0.953974
	1.571	2	0.283408	0.652830	0.803220	0.873324	0.911145	0.933824
	2.356	1	0.319200	0.666435	0.808038	0.875347	0.912208	0.934541
	3.141	1	0.181183	0.532522	0.713607	0.808068	0.862383	0.896278
	3.927	1	0.099144	0.415784	0.620217	0.737273	0.808014	0.853529
	4.712	1	0.052938	0.319200	0.532468	0.666435	0.751549	0.808038
0.95	0.628	7	0.354927	0.722284	0.848152	0.902809	0.931374	0.948268
	0.942	4	0.319275	0.692611	0.830351	0.891712	0.924072	0.943249
	1.257	3	0.260203	0.641135	0.797338	0.869691	0.908537	0.931762
	1.571	2	0.283408	0.652830	0.803220	0.873324	0.911145	0.933824
	2.356	2	0.101889	0.444136	0.652925	0.766233	0.832124	0.873366
	3.141	1	0.181183	0.532522	0.713607	0.808068	0.862383	0.896278
	3.927	1	0.099144	0.415784	0.620217	0.737273	0.808014	0.853529
	4.712	1	0.052938	0.319200	0.532468	0.666435	0.751549	0.808038
0.99	0.628	11	0.196370	0.599750	0.771971	0.851573	0.894295	0.919917
	0.942	6	0.180405	0.576413	0.756646	0.842048	0.888298	0.916092
	1.257	4	0.166118	0.552840	0.739361	0.830144	0.879948	0.910067
	1.571	3	0.150875	0.527473	0.719867	0.816137	0.869724	0.902396
	2.356	2	0.101889	0.444136	0.652925	0.766233	0.832124	0.873366
	3.141	2	0.032827	0.283579	0.509234	0.652973	0.743705	0.803313
	3.927	1	0.099144	0.415784	0.620217	0.737273	0.808014	0.853529

For illustration, Table 1 shows the minimal sample sizes and the corresponding OC under the G-L distribution for probability of acceptance $P^* = 0.75,\ 0.90,\ 0.95,\ 0.99,\ t/\mu_0 = 0.628, 0.942, 1.257, 1.571, 2.356, 3.141, 3.927, 4.712$ with $\alpha = 2,\ \lambda = 24$ and $c_1 = 0$. Also, Table 2 shows the minimal sample sizes for the first and second samples and the corresponding OC under the G-L distribution based on the DASP.

Table 2: OC values of the sampling plan $(n_1, n_2, c_1, c_2, t/\mu_o)$ for a given P^* under the G-L distribution with $\alpha = 2$,$\lambda = 24$.

P^*	t/μ_0	n_1	n_2	μ/μ_0					
				2	4	6	8	10	12
0.75	0.628	4	7	0.848369	0.990631	0.998570	0.999635	0.999873	0.999946
	0.942	2	4	0.861396	0.991078	0.998641	0.999661	0.999885	0.999953
	1.257	2	3	0.765844	0.980921	0.996893	0.999208	0.999732	0.999890
	1.571	1	3	0.788849	0.981478	0.996879	0.999194	0.999727	0.999888
	2.356	1	2	0.684457	0.962886	0.992926	0.998063	0.999323	0.999720
	3.141	1	2	0.451014	0.897839	0.976510	0.992930	0.997394	0.998884
	3.927	1	2	0.268918	0.800602	0.945222	0.981865	0.992924	0.996858
	4.712	1	2	0.150555	0.684457	0.897804	0.962886	0.984664	0.992926
0.90	0.628	6	10	0.679953	0.973035	0.995548	0.998829	0.999587	0.999823
	0.942	3	6	0.676258	0.970472	0.995071	0.998722	0.999560	0.999817
	1.257	2	4	0.679738	0.969199	0.994747	0.998636	0.999534	0.999808
	1.571	2	3	0.589491	0.952647	0.991389	0.997706	0.999209	0.999673
	2.356	1	3	0.483015	0.913417	0.981495	0.994672	0.998089	0.999195
	3.141	1	2	0.451014	0.897839	0.976510	0.992930	0.997394	0.998884
	3.927	1	2	0.268918	0.800602	0.945222	0.981865	0.992924	0.996858
	4.712	1	2	0.150555	0.684457	0.897804	0.962886	0.984664	0.992926
0.95	0.628	7	12	0.582576	0.958261	0.992786	0.998066	0.999312	0.999703
	0.942	4	7	0.537112	0.947237	0.990631	0.997510	0.999132	0.999635
	1.257	3	5	0.466427	0.927903	0.986439	0.996333	0.998721	0.999466
	1.571	2	4	0.481484	0.926504	0.985758	0.996102	0.998637	0.999433
	2.356	2	3	0.221542	0.806264	0.952686	0.985556	0.994656	0.997708
	3.141	1	2	0.451014	0.897839	0.976510	0.992930	0.997394	0.998884
	3.927	1	2	0.268918	0.800602	0.945222	0.981865	0.992924	0.996858
	4.712	1	2	0.150555	0.684457	0.897804	0.962886	0.984664	0.992926
0.99	0.628	11	16	0.336765	0.898012	0.980077	0.994387	0.997951	0.999104
	0.942	6	9	0.311881	0.882942	0.976567	0.993455	0.997661	0.999003
	1.257	4	6	0.299834	0.871915	0.973504	0.992542	0.997345	0.998878
	1.571	3	5	0.257481	0.841632	0.964906	0.989827	0.996335	0.998448
	2.356	2	3	0.221542	0.806264	0.952686	0.985556	0.994656	0.997708
	3.141	2	3	0.062506	0.589777	0.866411	0.952705	0.980960	0.991402
	3.927	1	2	0.268917	0.800602	0.945222	0.981865	0.992924	0.996858
	4.712	1	2	0.150555	0.684457	0.897804	0.962886	0.984664	0.992926

5 Minimum mean ratios to the specified life and producer's risk

The producer's risk is defined as the probability of rejection of a good lot ($\mu \geqslant \mu_0$). Based on the advised DASP and for a particular value of the producer's risk β, the aim is to find the minimum quality level μ/μ_0 that will guarantee the producer's risk to be at most β. Thus, μ/μ_0 is the lowest positive number for which $p = L\left(t;\mu\right) = L\left(\frac{t}{\mu_0}.\frac{\mu_0}{\mu}\right)$ the following inequality holds:

$$\sum_{i=0}^{c_1}\left(\begin{array}{c} n_1 \\ i \end{array}\right)p^i(1-p)^{n_1-i} + \sum_{j=c_1+1}^{c_2}\left(\begin{array}{c} n_1 \\ i \end{array}\right)p^i(1-p)^{n_1-i}\left(\sum_{i=0}^{c_2-j}\left(\begin{array}{c} n_2 \\ j \end{array}\right)p^j(1-p)^{n_2-j}\right) \geqslant 1-\beta. \tag{6}$$

For the proposed acceptance sampling plan $(n_1, n_2, c_1 = 0, c_2 = 2, t/\mu_o)$ at a specified consumer's confidence level P^*, the smallest values of μ/μ_0 satisfying 6 are summarized in Table 3 for $P^* = 0.75, 0.90, 0.95, 0.99$. Table 4 summarizes the values of the producer's risk based on the DASP for the G-L distribution for $c_1 = 0$ and $c_2 = 2$.

Table 3: Values of μ/μ_0 for the acceptability of a lot with producer's risk of 0.05 for the G-L distribution

P^*	t/μ_0							
	0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	2.703	2.656	3.156	3.110	3.692	4.922	6.154	7.384
0.90	3.445	3.514	3.543	3.945	4.664	4.922	6.154	7.384
0.95	3.826	4.055	4.398	4.428	5.916	4.922	6.154	7.384
0.99	4.812	5.001	5.143	5.496	5.916	7.887	6.154	7.384

Table 4: Producer's risk with respect to time of experiment for DASP based on the G-L distribution for $c_1 = 0$ and $c_2 = 2$.

P^*	t/μ_0	n_1	n_2	μ/μ_0					
				2	4	6	8	10	12
0.75	0.628	4	7	0.151631	0.009369	0.001430	0.000365	0.000127	0.000045
	0.942	2	4	0.138604	0.008922	0.001359	0.000339	0.000115	0.000045
	1.257	2	3	0.234156	0.019079	0.003107	0.000792	0.000268	0.000110
	1.571	1	3	0.211151	0.018522	0.003121	0.000806	0.000273	0.000112
	2.356	1	2	0.315543	0.037114	0.007074	0.001937	0.000677	0.000280
	3.141	1	2	0.548986	0.102161	0.023490	0.007070	0.002606	0.001116
	3.927	1	2	0.731082	0.199398	0.054778	0.018135	0.007076	0.003142
	4.712	1	2	0.849445	0.315543	0.102196	0.037114	0.015336	0.007074
0.90	0.628	6	10	0.320047	0.026965	0.004452	0.001171	0.000413	0.000177
	0.942	3	6	0.323742	0.029528	0.004929	0.001278	0.000440	0.000183
	1.257	2	4	0.320262	0.030801	0.005253	0.001364	0.000466	0.000192
	1.571	2	3	0.410509	0.047353	0.008611	0.002294	0.000791	0.000327
	2.356	1	3	0.516985	0.086583	0.018505	0.005328	0.001911	0.000805
	3.141	1	2	0.548986	0.102161	0.023490	0.007070	0.002606	0.001116
	3.927	1	2	0.731082	0.199398	0.054778	0.018135	0.007076	0.003142
	4.712	1	2	0.849445	0.315543	0.102196	0.037114	0.015336	0.007074
0.95	0.628	7	12	0.417424	0.041739	0.007214	0.001934	0.000688	0.000297
	0.942	4	7	0.462888	0.052763	0.009369	0.002490	0.000868	0.000365
	1.257	3	5	0.533573	0.072097	0.013561	0.003667	0.001279	0.000534
	1.571	2	4	0.518516	0.073496	0.014242	0.003898	0.001363	0.000567
	2.356	2	3	0.778458	0.193736	0.047314	0.014444	0.005344	0.002292
	3.141	1	2	0.548986	0.102161	0.023490	0.007070	0.002606	0.001116
	3.927	1	2	0.731082	0.199398	0.054778	0.018135	0.007076	0.003142
	4.712	1	2	0.849445	0.315543	0.102196	0.037114	0.015336	0.007074
0.99	0.628	11	16	0.663235	0.101988	0.019923	0.005613	0.002049	0.000896
	0.942	6	9	0.688119	0.117058	0.023433	0.006545	0.002339	0.000997
	1.257	4	6	0.700166	0.128085	0.026496	0.007458	0.002655	0.001122
	1.571	3	5	0.742519	0.158368	0.035094	0.010173	0.003665	0.001552
	2.356	2	3	0.778458	0.193736	0.047314	0.014444	0.005344	0.002292
	3.141	2	3	0.937494	0.410223	0.133589	0.047295	0.019040	0.008598
	3.927	1	2	0.731083	0.199398	0.054778	0.018135	0.007076	0.003142
	4.712	1	2	0.849445	0.315543	0.102196	0.037114	0.015336	0.007074

6 Use of tables and examples

In this section, we illustrate the results presented in the tables assuming that the lifetime of the product follows the G-L distribution with parameters $\alpha = 2$, $\lambda = 24$. Assume that the researcher wants to ensure that the true mean life is at least 1000 hours and the test is terminated at $t_0 = 1257$ with a confidence level of 0.95 for $c_1 = 0, c_2 = 2$. This leads to the ratio $t_0/\mu_0 = 1.257$ with corresponding sample sizes $n_1 = 3$ and $n_2 = 5$. This can be illustrated as follows. First, chose 3 products randomly from the lot and check them for 1257 hours. Accept the lot if no failure arises within 1257 hours. Reject the lot if more than two failures arise within the test. If just two failures are detected, draw a second sample of size 5 products and check them for 1257 hours. Accept the lot if there are no failures is observed from the second sample. Otherwise, the lot is rejected. Based on Tables 1 and 2, it is obvious that the smallest sample sizes of the first sample are less than the corresponding one gotten from the second sample. The OC function values can be obtained from Table 1 based on the single ASP and from Table 2 using the DASP for suggested ASP of the G-L distribution. In both tables, it can be noted that the OC values approach rapidly toward 1, and the DASP has more appropriate OC function values than the single ASP. For our example, Table 5 presents a comparison of the OC values of the single and double acceptance sampling plans. Figure 3 shows the OC function values for the SASP and DASP for $P^* = 0.95$,0.90,0.95,0.99 and $t/\mu_o = 1.257$ based on the G-L distribution.

Table 5: The OC function values based on single and double ASP for $P^* = 0.95$ and $t/\mu_o = 1.257$.

μ/μ_o	2	4	6	8	10	12
Single ASP	0.260203	0.641135	0.797338	0.869691	0.908537	0.931762
Double ASP	0.466427	0.927903	0.986439	0.996333	0.998721	0.999466

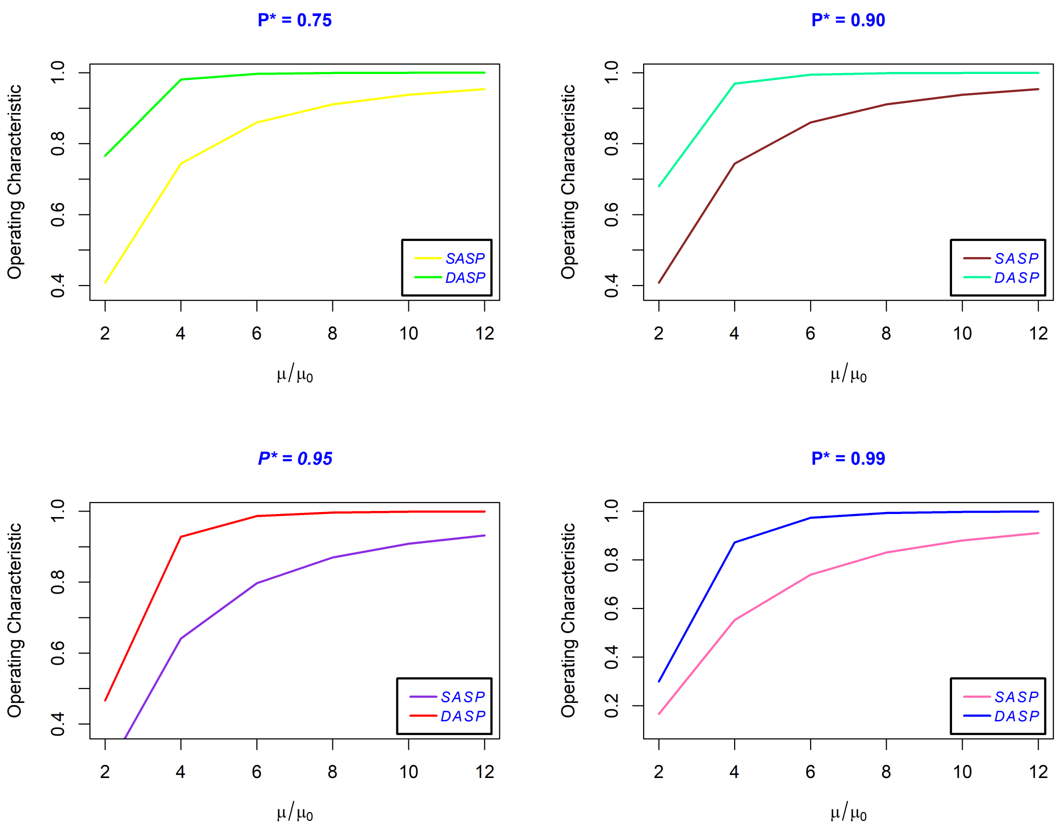

Figure 3: The OCF values for the SASP and DASP with $P^* = 0.75, 0.90, 0.95, 0.99$ and $t/\mu_o = 1.257$

From Table 3, the minimum ratio for $P^* = 0.95$ and $t/\mu_o = 1.257$ is 4.428. Therefore, the true mean essential of the product must be at least $4.428 \times 1000 = 4428$ hours. The producer's risk values reported in Table 4 are decreasing as the ratio μ/μ_0 values increase.

7 Conclusions

This article introduces a DASP using the mean as a quality parameter for the G-L distribution. For various values of the model parameters under a certain producer's risk and consumer's risk, the necessary design parameters are established, where the first and second samples' minimal sample sizes are derived as tables. The values of the operational characteristics and the minimal ratios of the mean life to the stipulated life are displayed. For fixed t_0/μ_0 , it is discovered that the minimum sample sizes are getting smaller as the confidence level gets smaller. Therefore, the researchers in the manufacturing sector are advised to use the proposed DASP for the G-L distribution. The suggested DASP for the G-L distribution can be investigated under ranked set sampling method Bouza Herrera & Al-Omari (2019), Al-Omari & Abdallah (2023), and Haq et al. (2015).

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