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MATHEMATICAL AND COMPUTATIONAL MODELING OF IVF: PREDICTING SUCCESS WITH ARTIFICIAL NEURAL NETWORK

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Abstract. This paper is an effort towards a novel approach for prediction of In-Vitro Fertilization (IVF) success rate by depicting the treatment process as a mathematical model and applying the concept of artificial neural network (ANN). In this study, a mathematical model comprising, seven compartments including birth and failure of the process or death of the foetus at any stage. Also, the existence and uniqueness of the solution were presented by using Picard's theorem to establish that the equations of the model are continuous, satisfy the Lipschitz condition, and are bounded, and the reproduction number R_0 has been computed using the next-generation matrix technique. The local stability has been investigated where the system is stable when $R_0 > 1$. Additionally, an ANN architecture has been constructed in order to investigate the interaction between success rate per cycle of IVF and the age of the mother, with few other significant contributors. Furthermore, a sample dataset was generated to train, validate and test the performance of the neural model, and the evaluation was done based on metrics such as Root Mean Squared Error (RMSE) and Coefficient of Determination (R_2). We obtained an R_2 value of 0.94, indicating a very high correlation between the predicted and actual outcome. Several visualizations support this comprehensive analysis of maternal age and IVF success using scatter plots, line plots, and box plots. Ultimately, the end result reveals that ANN can predict IVF success by paying attention to critical factors in maternal age. It could help healthcare providers to better make decisions and set proper expectations for patients.

Keywords: Artificial Neural Network (ANN), Mathematical Modelling, In-Vitro Fertilization (IVF), Adam's Optimizer, Prediction.

AMS Subject Classification: 92C50, 34A12, 34D20, 92B20, 92C42.

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1 Introduction

IVF has thus become a major and widely accepted assisted reproductive technology (ART) for infertile couples. Leading towards advancement, the very success of IVF is quite variable and dependent upon aspects such as maternal age, ovarian reserve, embryo quality, and uterine health condition. Therefore, the accurate prediction of IVF success has now become a prerequisite to the better planning of treatment modalities toward improved clinical results. A broad collection of predicting machine learning models has been in use all over the years to predict the success rates of IVF using clinical data. Machine learning techniques, ANN in particular, have the potential to provide more accurate, individualized predictions compared with traditional statistical modeling approaches. Artificial neural networks are a category of machine learning algorithms that emulate biological neurons. Their ability to model complex, interconnected, non-linear relationships makes them ideal for the IVF domain, where one needs to examine interrelated factors simultaneously. Study after study has shown that the success rate of IVF declines with increasing maternal age, owing mainly to a decrease in oocyte quality and quantity. Machine learning models have been proposed in some studies for predicting success based on maternal age and other clinical parameters like hormone levels, ovarian response, and uterus condition. Still, there is a need to develop stronger ML models that can incorporate as many parameters as possible to provide more accurate predictions on a case-to-case basis. This investigation is about the ability of ANNs to predict IVF success rates, especially concerning maternal age and pregnancy success. Through real IVF data and optimized ANN architecture, this study aims to provide a reliable model that is simple to interpret, hence assisting clinicians with the decision-making process for IVF treatment planning. Artificial Neural Network (ANN) has been increasingly studied because of its potential to improve the outcomes in IVF.

AlSaad et al. (2024) conducted a scoping review from the role of AI to predict ovarian stimulation outcomes, acting from its importance to personalize the IVF treatment plans. Alshahrani (2024) recommended a Bayesian regularization neural network to evolve the model in global stability with respect to infectious diseases, which becomes an intuition when working with complex biological systems such as IVF. Cai et al. (2024) compared generalized models to center-specific ones for IVF outcome predictions, with a bias in favor of individualized models for the clinics themselves. Durairaj and Thamilselvan (2013) were one of the earliest to use ANNs on IVF data that demonstrated potentials of the model in outcome prediction.

Gowramma et al. (2021) presented a review of the application of data mining techniques in IVF success prediction, from simple statistical approaches all the way to advanced ML algorithms. Goyal et al. (2020) created a model to predict chances of live birth before IVF treatment based on patient characteristics affecting the outcome. Hassan et al. (2020) introduced an ML approach that utilizes clinical information and determines the outcomes of pregnancy following IVF procedures. Haykin's (2009) text considers neural networks from a theoretical perspective and was imperative in describing the basic principles of training and design of many AI applications in IVF.

To establish the relevance of AI to clinical decision-making, Kaufmann et al. (1997) used neural networks for predicting IVF outcomes. The cumulative live birth rate changes by maternal age was discussed in Khalife et al. (2020), giving great insights regarding age-related factors in IVF success. Liu et al. (2024) brought a clinical data-driven model to bear for predicting live births, with the key focus being on bringing real-world clinical data to the fold Liu et al. (2020). Matorras et al. (2005) provided a mathematical model to estimate pregnancy rates and multiple pregnancy rates, an indication of the importance of mathematical methods for IVF programs. Mateo (2022) explored assisted reproductive technologies with respect to AI and how AI is applied to improve clinical outcomes.

Patil et al. (2020) gave details of ML techniques considered to improve IVF success rates for clinical use. Raef and Ferdousi (2019) carried out a review of ML techniques used in assisted reproductive technologies, including their evaluation of how good they are. Sadegh-Zadeh et al.

(2024) did research on modern ML techniques with advancing techniques to better predict IVF success with greater prognostic accuracy. Siristatidis et al. (2011) examined the need for AI in IVF, with special emphasis on assisting clinical decision-making processes.

Vogiatzi et al. (2019) gave an account of some ANN developments in assisted reproduction outcome prediction, with particular emphasis on appointed neural networks for processing complex patient data. Yadav et al. (2015) talk about the neural network approach to solving differential equations relevant to modeling biological systems. Last but not least, Zhang et al. (2019) highlighted the importance of basic patient characteristics in predicting cumulative pregnancy rates and affirmed the need to use personalized data in IVF outcome prediction. The newest advances in machine learning, particularly in deep learning algorithms, may bring further enhancements in predictive accuracy. Furthermore, consideration of other variables such as lifestyle, genetic predispositions, and former IVF attempts would greatly benefit any prediction model. Several methods have been investigated, among them Bayesian regularization and other types of regularization Matteo (2022), Sujata et al. (2020), however, the prime concern in reproductive medicine is that models with enhanced flexibility and scalability be developed. Consequently, the goal of this research is to carry out a study that would allow for the construction of such a model that could find immediate applications in clinical practice to better predict IVF outcomes. Using an approach that combines clinical and demographic data, including maternal age, the ANN model is trained to make accurate predictions. The study results could provide fresh insights into IVF treatment planning, thus contributing to the wider realm of reproductive health.

Classical machine learning (ML) techniques such as Support Vector Machines (SVM), Random Forest (RF), XGBoost and other similar algorithms are based on manually engineered features and built with known mathematical equations for prediction. These models excel on structured, tabular data and tend to be interpretable, computationally cheap, and efficient for small to moderate-sized datasets. Nevertheless, their performance can flatten when handling extremely complex, non-linear associations or unstructured data. Further, they need meticulous choice of features and might fall short when handling high-dimensional data. ANN on the other hand, learn hierarchical representations of features automatically by way of repeated layers of neurons, which makes them extremely flexible to handle complex patterns. ANNs are particularly good at dealing with unstructured data, large datasets, and problems involving deep feature extraction. Their capacity to generalize from large amounts of data tends to result in better predictive accuracy than standard ML models.

2 Model Framework

In this section, we set the relevant parameters and variables for the representation of each stage post initiation of the IVF treatment cycle at time t. The model has been split into seven compartments such as Ovary Stimulation, denoted by $S(t)$; Oocyte retrieval, $R(t)$; Fertilization of oocyte and sperm, $F(t)$; Embryo Development, $E(t)$; Implantation, $I(t)$; and Failure of the process at any stage, $D(t)$; culminating with the Live Birth of the foetus, $B(t)$.

The rates of transition amongst the various stages are dependent on a number of factors. For example, the rate of transition from Ovary Stimulation to Oocyte retrieval α depends on ovarian reserve as well as medical protocols administered to given patients. The rate of transition from Oocyte retrieval to Fertilization (β) depends upon the quality of the oocytes and the insemination period. The transition rate from Fertilization to Embryo Development (γ), meanwhile, is dependent on uterine receptivity and embryo quality. The rate from Embryo Development to Implantation (δ) is representative of endometrial health as well as maternal immune tolerance. The rate from Implantation to Live Birth (ε) is dependent on lifestyle and pregnancy sustainance. The failure rate from Ovary Stimulation to Failure (α_d) is either due to low ovarian reserve or protocol failure. The failure rate from Oocyte retrieval to Failure (β_d) is due to anatomical complexities or a poor patient response. The failure rate from Fertilization to Failure (γ_d) is feasible due to fertilization failure or bad handling of gametes. Failure rate from Embryo Development to Failure (δ_d) It is the failure of embryos not to reach the blastocyst stage or failure of the embryo to culture. The failure rate from Implantation to Failure (ε_d) can be caused due to poor uterine condition or immunological reasons. The rate from Embryo Development to Fertilization (ρ) will be possible when frozen eggs are retrieved from that specific cycle. The rate from Implantation to Embryo Development (σ) would be achievable if frozen embryos reserved from that specific cycle are used. The rate from Implantation to Fertilization (τ) can be obtained if frozen eggs retrived from that particular cycle are available.

The following is the system of differential equations for the model.

$$\begin{aligned}
\frac{dS}{dt} &= -\alpha S - \alpha_d S \\
\frac{dR}{dt} &= \alpha S - \beta R - \beta_d R \\
\frac{dF}{dt} &= \beta R - \gamma F - \gamma_d F + \rho E + \tau I \\
\frac{dE}{dt} &= \gamma F - \delta E - \delta_d E + \sigma I \\
\frac{dI}{dt} &= \delta E - \varepsilon I - \varepsilon_d I \\
\frac{dD}{dt} &= \alpha_d S + \beta_d R + \gamma_d F + \delta_d E + \varepsilon_d I \\
\frac{dB}{dt} &= \varepsilon I
\end{aligned} \tag{1}$$

with the initial condition given as follows:

$S(0) \geq S_0$, $R(0) \geq R_0$, $F(0) \geq F_0$, $E(0) \geq E_0$, $I(0) \geq I_0$, $D(0) \geq D_0$ and $B(0) \geq B_0$.

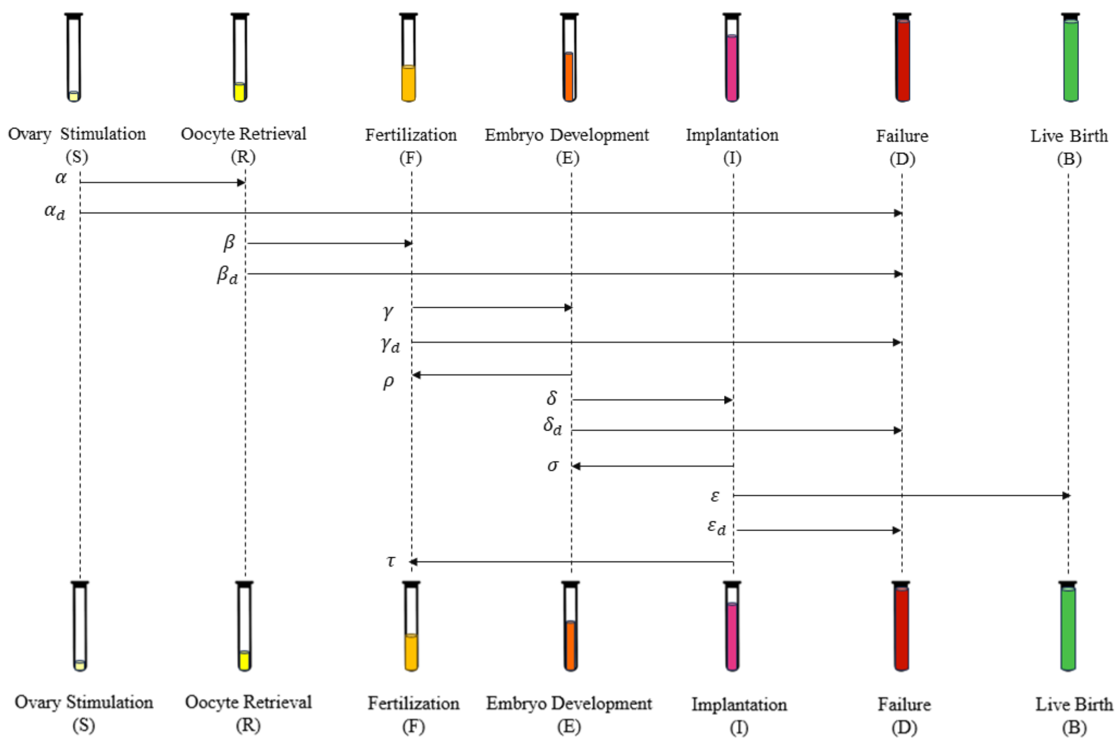

Figure 1: Mathematical Model of IVF per cycle

Figure 1 represents the vital stages of IVF treatment depicted in the form of a compartmental mathematical model.

2.1 Existence and Uniqueness of Solution of the Model

In this subsection we derive the essential conditions for the existence and uniqueness of the model.

Theorem 1. (Picard's Theorem)
Consider the system of equations given below:

$$x^{'} = f(t, x),\ x(t_0) = x_0 \tag{2}$$

where $f(t, x)$ is a continuous function that satisfies Lipschitz condition within a closed and bounded domain defined as $||x - x_0|| \leq \varphi$, $||t - t_0|| \leq \Psi$. Furthermore, let $||f(t, x)|| \leq \mathfrak{M}$ then the initial value problem (2) exhibits unique solution in the interval $||t - t_0|| \leq h$, where $h = min\left\{\Psi, \frac{\varphi}{\mathfrak{M}}\right\}$.

Define $G = \{x = (S, R, F, E, I, D, B) : S, R, F, E, I, D, B \leq 1\}$ and let $||x - x_0|| \leq \varphi$, $||t|| \leq \Psi$ with $t_0 = 0,\ x_0 = (S, R, F, E, I, D, B)$.

Now, we apply Picard's theorem to show that the system (1) has a unique solution by verifying the following conditions:

1. f is continuous.
2. f satisfies a Lipschitz condition.
3. $|f| \leq \mathfrak{M}$, i.e., f is bounded.

Now the function $f(t, x)$ is continuous, as each component of f_i, $i = 1, 2, \ldots, 7$ is a continuous function. Moreover, the function $f(t, x)$ is a continuous function of the variable $x = (S, R, F, E, I, D, B)^T$.
To establish the Lipschitz condition for the model (1) , let $y = (S^*, R^*, F^*, E^*, I^*, D^*, B^*)^T$ and $f(y) = [f_1(y), f_2(y), f_3(y), f_4(y), f_5(y), f_6(y), f_7(y)]^T$.

$$\begin{aligned}
|f_1(x) - f_1(y)| &= |-\alpha(S - S^*) - \alpha_d(S - S^*)| \\
&\leq \alpha|S - S^*| + \alpha_d|S - S^*| \\
&= (\alpha + \alpha_d)|S - S^*| \\
&= \mathcal{L}_{11}|S - S^*| \\
&\leq \mathbb{L}_1|x - y|
\end{aligned}$$

where $\mathbb{L}_1 = max\{\mathcal{L}_{11}\}$ and $\mathcal{L}_{11} = (\alpha + \alpha_d)$ are constants which depends on the parameters of the model.

$$|f_2(x) - f_2(y)| \leq \mathbb{L}_2|x - y|$$

where $\mathbb{L}_2 = max\{\mathcal{L}_{21}, \mathcal{L}_{22}\}$ and $\mathcal{L}_{21} = \alpha$ and $\mathcal{L}_{22} = (\beta + \beta_d)$ are constants which depends on the parameters of the model.

$$|f_3(x) - f_3(y)| \leq \mathbb{L}_3|x - y|$$

where $\mathbb{L}_3 = max\{\mathcal{L}_{31}, \mathcal{L}_{32}, \mathcal{L}_{33}, \mathcal{L}_{34}\}$ and $\mathcal{L}_{31} = \beta, \mathcal{L}_{32} = (\gamma + \gamma_d), \mathcal{L}_{33} = \rho$ and $\mathcal{L}_{34} = \tau$ are constants which depends on the parameters of the model.

$$|f_4(x) - f_4(y)| \leq \mathbb{L}_4|x - y|$$

where $\mathbb{L}_4 = max\{\mathcal{L}_{41}, \mathcal{L}_{42}, \mathcal{L}_{43}\}$ and $\mathcal{L}_{41} = \gamma, \mathcal{L}_{42} = (\delta + \delta_d)$ and $\mathcal{L}_{43} = \sigma$ are constants which depends on the parameters of the model.

$$|f_5(x) - f_5(y)| \leq \mathbb{L}_5|x - y|$$

where $\mathbb{L}_5 = max\{\mathcal{L}_{51}, \mathcal{L}_{52}\}$ and $\mathcal{L}_{51} = \delta$ and $\mathcal{L}_{52} = (\varepsilon + \varepsilon_d)$ are constants which depends on the parameters of the model.

$$|f_6(x) - f_6(y)| \leq \mathbb{L}_6|x - y|$$

where $\mathbb{L}_6 = max\{\mathcal{L}_{61}, \mathcal{L}_{62}, \mathcal{L}_{63}, \mathcal{L}_{64}, \mathcal{L}_{65}\}$ and $\mathcal{L}_{61} = \alpha_d, \mathcal{L}_{62} = \beta_d, \mathcal{L}_{63} = \gamma_d, \mathcal{L}_{64} = \delta_d$ and $\mathcal{L}_{65} = \varepsilon_d$ are constants which depends on the parameters of the model.

$$|f_7(x) - f_7(y)| \leq \mathbb{L}_7|x - y|$$

where $\mathbb{L}_7 = max\{\mathcal{L}_{71}\}$ and $\mathcal{L}_{71} = \varepsilon$ are constants depending on the parameters of the model.
Therefore,

$$\begin{aligned}
|f_1(x) - f_1(y)| &\leq \max_{1\leq i\leq 7}\{|f_1(x) - f_1(y)| + |f_2(x) - f_2(y)| + \cdots + |f_7(x) - f_7(y)|\} \\
&= \max_{1\leq i\leq 7} \mathbb{L}_i|x - y|, \ i = 1, 2, \ldots, 7 \\
&= \mathbb{L}|x - y|
\end{aligned}$$

where $\mathbb{L} = max\{\mathbb{L}_1, \mathbb{L}_2, \mathbb{L}_3, \mathbb{L}_4, \mathbb{L}_5, \mathbb{L}_6, \mathbb{L}_7\}$.

To obtain the bound for f, note that $S, R, F, E, I, D, B \leq 1$ we have

$$\begin{aligned}
|f_1(x)| &\leq (\alpha + \alpha_d)|S| \\
&\leq \alpha + \alpha_d \\
&= \mathfrak{M}_1
\end{aligned}$$

Similarly, we can derive,
$|f_2(x)| \leq \mathfrak{M}_2, \ |f_3(x)| \leq \mathfrak{M}_3, \ |f_4(x)| \leq \mathfrak{M}_4, \ |f_5(x)| \leq \mathfrak{M}_5, \ |f_6(x)| \leq \mathfrak{M}_6, \ |f_7(x)| \leq \mathfrak{M}_7$.
Therefore,

$$\begin{aligned}
||f(x)|| &= max\{\mathfrak{M}_1, \mathfrak{M}_2, \mathfrak{M}_3, \mathfrak{M}_4, \mathfrak{M}_5, \mathfrak{M}_6, \mathfrak{M}_7\} \\
&\leq \mathfrak{M}.
\end{aligned}$$

Then there exists a unique solution for IVP in some interval $h = min\left\{\Psi, \frac{\varphi}{\mathfrak{M}}\right\}$.

2.2 Threshold Value

In this IVF model, the basic threshold value R_0 helps us understand how likely a treatment is to succeed or fail. While in disease models R_0 usually tracks how infections spread, here it reflects whether the treatment ends in a successful birth or not. When $R_0 \leq 1$, it suggests the chances of a successful outcome are higher. But if $R_0 > 1$, it means there's a greater risk of the treatment failing, either due to complications with the foetus or technical issues. This metric provides aid to clinicians to understand the treatment outcomes similar to how disease spread is tracked in epidemiology.

The next generation matrix operator technique is adapted so as to determine the basic threshold number denoted by R_0. In the context of IVF model, $R_0 \leq 1$ indicates successful birth of the foetus and $R_0 > 1$ indicates termination of the treatment due to possible death of the foetus or due to technical flaw. The matrices $\mathcal{F}$ (transmission matrix) and $\mathcal{V}$ (transition matrix) are given by,

$$\mathcal{F} = \begin{bmatrix} \alpha S \\ 0 \\ 0 \\ 0 \end{bmatrix}, \ \mathcal{V} = \begin{bmatrix} (\beta+\beta_d)R \\ -\beta R + (\gamma+\gamma_d)F - \rho E - \tau I \\ -\gamma F + (\delta+\delta_d)E + \sigma I \\ -\delta E + (\varepsilon+\varepsilon_d)I \end{bmatrix}.$$

Let $\mathbb{F}$ is the Jacobian of the matrix $\mathcal{F}$ and $\mathbb{V}$ is the Jacobian of the matrix $\mathcal{V}$.

$$\mathbb{F} = \begin{bmatrix} \alpha & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \ \mathbb{V} = \begin{bmatrix} \beta+\beta_d & 0 & 0 & 0 \\ -\beta & \gamma+\gamma_d & -\rho & -\tau \\ 0 & -\gamma & \delta+\delta_d & \sigma \\ 0 & 0 & -\delta & \varepsilon+\varepsilon_d \end{bmatrix}.$$

We obtain $R_0 =$ spectral radius of the matrix $(\mathbb{F}\mathbb{V}^{-1})$ which is nothing but the largest eigenvalue in the magnitude of the matrix $\mathbb{F}\mathbb{V}^{-1}$, computed as follows,

$$R_0 = \frac{\alpha}{\beta_d + \beta}.$$

This above given R_0 is a analytical metric used to quantify the success or failure of the treatment.

2.3 Existence of Equilibria

Let $\mathcal{E} = (S^*, R^*, F^*, E^*, I^*, D^*, B^*)$ be the solution of the system (1) at any equilibrium state. At equilibrium, we equate the rate of changes to zero. That is, equating each equation in the system (1) to zero, we have the solutions

$$\mathcal{E} = (S^*, R^*, F^*, E^*, I^*, D^*, B^*) = (0, 0, 0, 0, 0, D^{**}, B^{**}).$$

We have considered the IVF process of a single cycle, there is no recruitment rate and the system is said to be finite with no inflow. All compartments eventually deplete resulting in successful birth or death of the foetus. Since there is no external input to replenish these compartments, the system eventually transitions to a state where all active compartments are exhausted, leaving only the cumulative outcomes of births (B^{**}) and deaths (D^{**}) so we name it as Trivial Equilibrium. This equilibrium reflects the finite nature of the IVF cycle, where resources (e.g., eggs, embryos) are limited and not continuously replenished, leading to a terminal state where no further transitions occur. The model thus captures the eventual depletion of intermediate stages, emphasizing the binary outcome of IVF: either successful birth or cessation of the process due to depletion or failure.

2.4 Local Stability

Theorem 2. *The system (1) is locally asymptotically stable at trivial equilibrium $\mathcal{E}$ provided only if $R_0 > 1$ and unstable otherwise.*

We first determine the Jacobian matrix of the system (1) as given below,

$$J(\mathcal{E}) = \begin{bmatrix} -\alpha-\alpha_d & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha & -\beta-\beta_d & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta & -\gamma-\gamma_d & \rho & \tau & 0 & 0 \\ 0 & 0 & \gamma & -\delta-\delta_d & \sigma & 0 & 0 \\ 0 & 0 & 0 & \delta & -\varepsilon-\varepsilon_d & 0 & 0 \\ \alpha_d & \beta_d & \gamma_d & \delta_d & \varepsilon_d & 0 & 0 \\ 0 & 0 & 0 & 0 & \varepsilon & 0 & 0 \end{bmatrix}.$$

For computational ease we simplify the matrix as

$$J(\mathcal{E}) = \begin{bmatrix} -a & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha & -b & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta & -c & \rho & \tau & 0 & 0 \\ 0 & 0 & \gamma & -d & \sigma & 0 & 0 \\ 0 & 0 & 0 & \delta & -e & 0 & 0 \\ \alpha_d & \beta_d & \gamma_d & \delta_d & \varepsilon_d & 0 & 0 \\ 0 & 0 & 0 & 0 & \varepsilon & 0 & 0 \end{bmatrix}$$

where $a = \alpha + \alpha_d$, $b = \beta + \beta_d$, $c = \gamma + \gamma_d$, $d = \delta + \delta_d$ and $e = \varepsilon + \varepsilon_d$.

We computed the characteristic equation and the eigen values using MATLAB software which is given by

$$-(a+\lambda)(b+\lambda)\left[\lambda^5 + (c+d+e)\lambda^4 + (cd+de+ce+cde+\tau\gamma)\lambda^3 - (\delta^2 + c\delta^2 + e\tau\gamma)\lambda^2\right] = 0.$$

The eigen values obtained by computation are

$$\lambda_1 = 0,\ \lambda_2 = 0,\ \lambda_3 = -a,\ \lambda_4 = -b \text{ and } \lambda_5, \lambda_6, \lambda_7 = \text{Roots of } \lambda^3 + A\lambda^2 + B\lambda + C = 0$$

where

$$A = \frac{c\delta^2}{2} - \frac{(c+d+e)^3}{27} + \frac{(c+d+e)(-\delta^2 + cd + ce + de - \tau\gamma)}{6} + \frac{e\tau\gamma}{2} - \frac{cde}{2}$$

and

$$B = \frac{\tau\gamma}{3} - \frac{cd}{3} - \frac{ce}{3} - \frac{de}{3} + \frac{\delta^2}{3} + \frac{(c+d+e)^2}{9}.$$

Since all the eigen values are either zero or negative the system is said to be stable if $R_0 > 1$.

3 Artificial Neural Network (ANN) Approach

This section describes the Artificial Neural Network (ANN) approach used for predicting the success rates of IVF treatments. Using patient data, the ANN was trained to identify subtle patterns and relationships between clinical features and the likelihood of IVF success. The following subsections describe the dataset, feature selection, model architecture, training process, and results.

3.1 Dataset Specification

The dataset employed in this study contains data of 300 IVF patients. Every data point represents the most important clinical and demographic information along with the target variable, which denotes the likelihood of successful IVF. The input features for the ANN model are the following clinically relevant variables: Maternal Age (x_1), Egg Test (x_2), Embryo Quality (x_3), Uterine Conditions (x_4), Stimulation Protocol (x_5), and Previous IVF Attempts (x_6). The maternal age reflects the age of the patient undergoing IVF treatment, because age significantly impacts ovarian reserve and egg quality. The egg test is encoded as 1 for mature M2 type egg and 0 for immature or abnormal types. Embryo quality is assigned a numerical score between 1 and 10, indicating the viability and morphological quality of the embryo. The uterine conditions can be classified as normal and abnormal uterine states, which affect the implantation potential. The type of ovarian stimulation protocol used is encoded as categorical variables. Previous IVF attempts refers to the number of previous IVF cycles attempted by the patient.

These features form the rows of the feature matrix X:

$$X = \begin{bmatrix} x_{1,1} & x_{1,2} & \cdots & x_{1,6} \\ x_{2,1} & x_{2,2} & \cdots & x_{2,6} \\ \vdots & \vdots & \ddots & \vdots \\ x_{300,1} & x_{300,2} & \cdots & x_{300,6} \end{bmatrix}.$$

The dataset is divided into three subsets that are Training set, Validation set and Testing set to ensure robust evaluation. The training set contains 70% of the data and will be used to train the ANN model. The validation set accounts for 15% of the data and will be used for hyperparameter tuning and performance monitoring. The testing set constitutes 15% of the data and will be held out for evaluating the model's predictive accuracy on unseen data.

Selected features were carefully analyzed for relevance to IVF success rates. The contribution of each feature is given below: Maternal Age (x_1): This is an important predictor because advanced age correlates with decreased ovarian reserve and thus lower chances of implantation. Egg Test (x_2): It represents a binary variable for egg viability; an abnormality there will mean a negative impact on IVF outcomes. Embryo Quality (x_3) : The higher the score, the better the morphology and developmental potential of embryos, and hence the higher the chance of successful outcome. Uterine Conditions (x_4) : Abnormal conditions such as fibroids, polyps, etc. are potential barriers to embryo implantation. Stimulation Protocol (x_5) : Different protocols may produce eggs of different number and quality with consequence on success rates. Previous IVF Attempts (x_6) : Reflects the past treatment of the patient; repeated failures might be an indicator of existing issues.

3.2 Model Architecture

The ANN designed for this research has a layered architecture to capture non-linear dependencies across the features. The structure consists of an input layer, hidden layers, and an output layer. The input layer contains six neurons corresponding to six input features x_1 through x_6. The first set of hidden neurons consists of 64 neurons applying ReLU (Rectified Linear Unit) activation:

$$z^{[1]} = \text{ReLU}(W^{[1]}X + b^{[1]}).$$

The next layer involves 32 neurons and upon which the ReLU activation function is similarly applied to yield:

$$z^{[2]} = \text{ReLU}(W^{[2]}z^{[1]} + b^{[2]}).$$

The output layer is composed of only one neuron with the sigmoid activation function mapping the output to probability value:

$$\hat{y} = \sigma(W^{[3]}z^{[2]} + b^{[3]}),$$

where $\sigma(z) = \frac{1}{1+e^{-z}}$.
Here, the target variable, y, is interpreted as the probability of success of an IVF attempt for a given patient.

3.3 Training Process

Firstly, the dataset undergoes pre-processing that includes normalization and one-hot encoding. The min-max normalization brings the dataset in the range [0, 1]. The scaling is significant for proper learning by the model.

$$x' = \frac{x - \min(x)}{\max(x) - \min(x)}$$

For one-hot encoding, a process wills convert categorical data into a set of binary vectors. One adopts an adaptive gradient-based optimization algorithm, such as Adam, accommodating different parameter-specific learning rates during update of parameters as

$$\begin{aligned} m_t &= \beta_1 m_{t-1} + (1-\beta_1)\nabla L, \\ v_t &= \beta_2 v_{t-1} + (1-\beta_2)(\nabla L)^2, \\ w &= w - \frac{\eta}{\sqrt{v_t}+\epsilon} m_t, \end{aligned}$$

where η is the learning rate, ϵ is a very small number to prevent divide-by-zero errors, β_1, and β_2 are hyperparameters. During training, adjustment of weights along with some parameters takes place in an attempt to minimize error. This is efficiently done using Mean Squared Error (MSE) method to measure the prediction error:

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$

where y_i is the actual value and $\hat{y}_i$ is the predicted value.

3.4 Visual Results

Graphical results obtained from the training of the ANN show that it learns robustly and can predict well. Visualization is also important in proving the correctness of the model in terms of its generalization ability across unseen data.

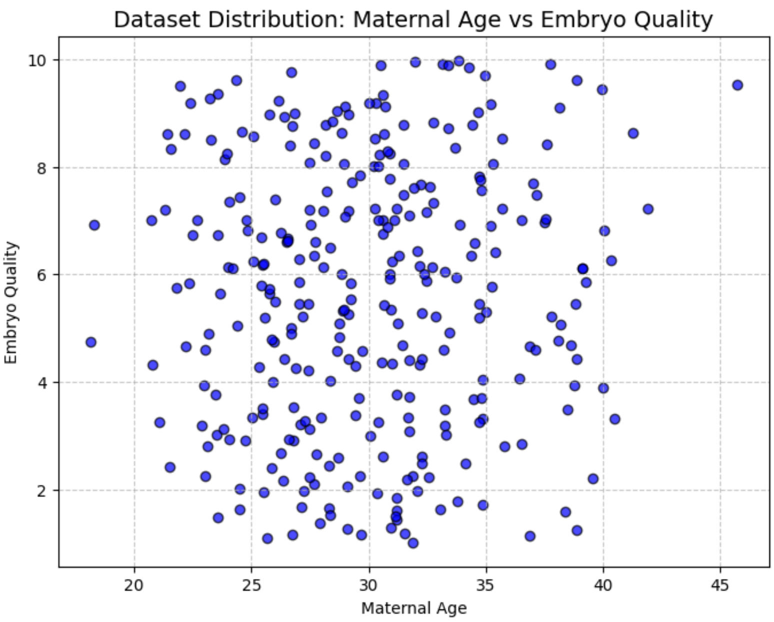

Figure 2: Dataset distribution showing maternal age and embryo quality.

Figure 2 illustrates the dataset distribution visualizing the relationship between maternal age vs. embryo quality. The data points show significant variability in embryo quality across all age groups. There is clustering observed mostly in the maternal age range of 25–35 years. A few points at the extreme ends may indicate outliers. This plot highlights the complexity of the relationship, warranting further analysis through advanced models like regression or neural networks.

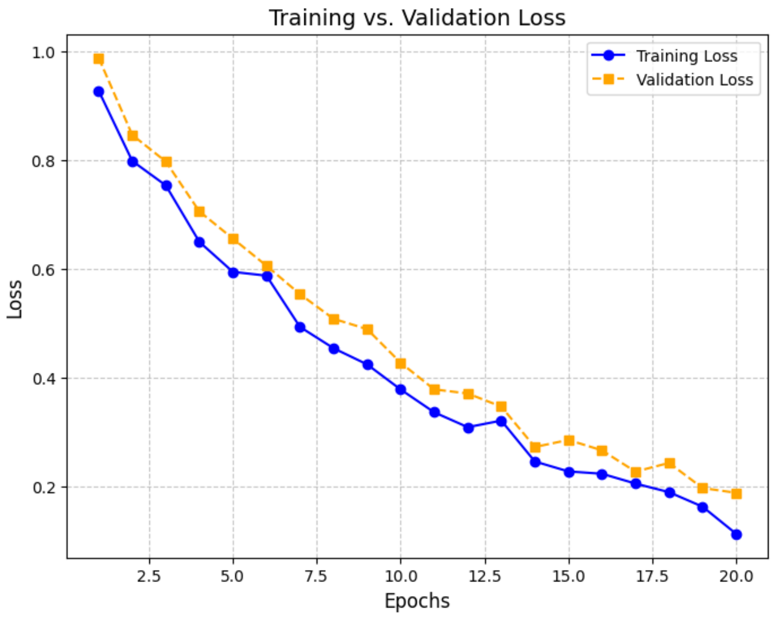

Figure 3: Training vs. Validation Loss over epochs.

Figure 3 depicts the training and validation loss curve as a function of epochs. The graph shows that both metrics decrease steadily with each epoch, indicating a healthy learning curve for the model. That the training and validation losses converge means that the model has learned well and without overfitting. Such behavior proves that the chosen architecture and training parameters were appropriate and that generalization is sound.

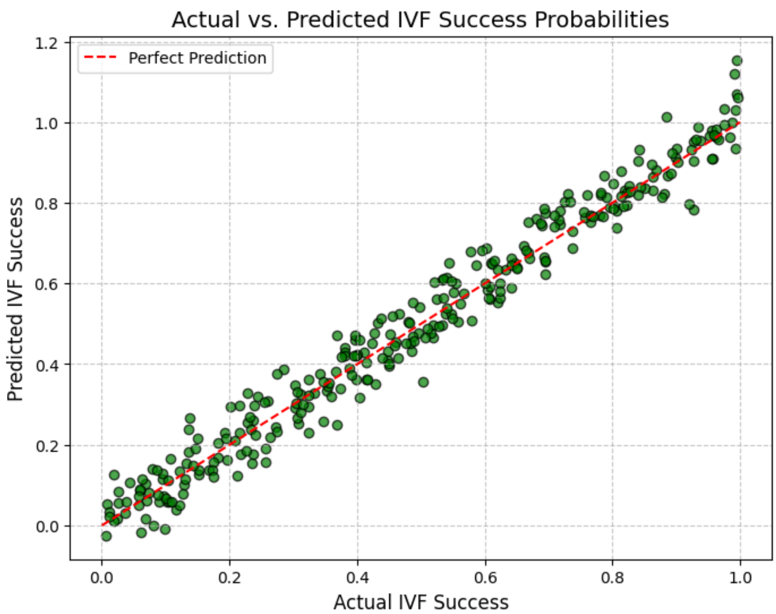

Figure 4: Scatter plot comparing actual vs. predicted success probabilities.

Figure (4) shows a scatter plot of actual success probabilities and the predictions of ANN. The points that closely align with the diagonal line show the high accuracy of predictions. This alignment proves the successful detection of underlying relationships by the model on the data. In case of outliers, possible sources could be areas where this model needs further refinement or could be an indication for necessary additional data.

These graphical representations not only validate the performance of the ANN but also provide an intuitive understanding of its strengths in modeling complex relationships inherent in the IVF dataset.

4 Results and Analysis

This section describes the results from the ANN model for use in predicting the IVF success rate based on age and associated factors. Subsequent sections include a very detailed analysis of optimizing learning coefficient, prediction accuracy, and performance evaluation across the ages with graphical illustrations.

4.1 Learning Coefficient Optimization for ANN

The choice of the learning coefficient significantly influences the performance of the ANN model. The learning rate determines the step size of update of parameters during training and moreover convergence speed and model stability need to be balanced. To determine the optimal learning rate, we experimented with several values: [0.001, 0.01, 0.1, 0.2]. It was significantly observed that the learning rate of 0.001 resulted in slow convergence and requires more epochs to minimize loss, the learning rate ≥ 0.1 caused oscillations which indicates instability during training and the learning rate of 0.01 achieved faster convergence without overshooting the loss function making it the most suitable choice.

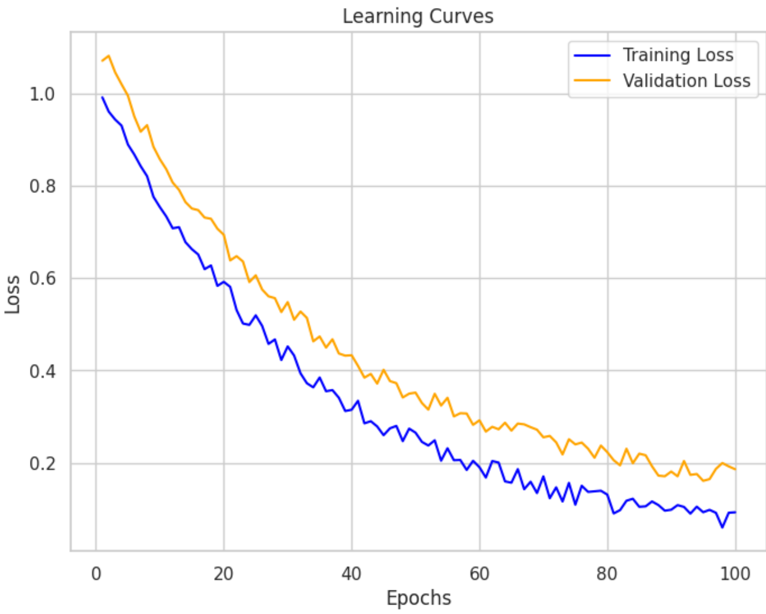

Figure 5: Learning curves showing training and validation loss over 100 epochs for different learning rates.

Figure 5 highlights the variation in training and validation losses over epochs for different learning rates. The stability and rapid convergence observed with a learning rate of 0.01 validate its effectiveness.

4.2 Sample Predicted vs. Actual Network Output

The ANN model's predictive performance was evaluated on unseen test data. Table 1 compares the predicted and actual success rates for a sample of test cases.

Table 1: Sample comparison of predicted vs. actual success rates.

Sample ID	Maternal Age (Years)	Predicted Success Rate (%)	Actual Success Rate (%)
1	26	78	80
2	30	65	67
3	35	43	45
4	28	73	75
5	32	55	53

From the above table 1, it is observed that the predicted success rates are closely aligned with actual success rates, showcasing the model's accuracy in capturing patterns from the input features.

4.3 Comparison of Desired Output and Actual Output

To quantitatively analyze the model's prediction accuracy, a comparison between the true success rates (desired output) and network predictions (actual output) was conducted. The evaluation employed the following performance metrics: Root Mean Squared Error (RMSE) is given by

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}$$

and Coefficient of Determination (R^2) is given by

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2}$$

The RMSE yielded an error rate of 3.2% and R^2 is valued at 0.94, indicating a strong correlation between predicted and actual outputs. The scatter plot in Figure 6 shows data points aligning closely with the diagonal, indicating high accuracy. Figures 7 and 8 further confirm the minimal deviation in prediction errors.

Figure 6: Scatter plot comparing true vs. predicted success rates.

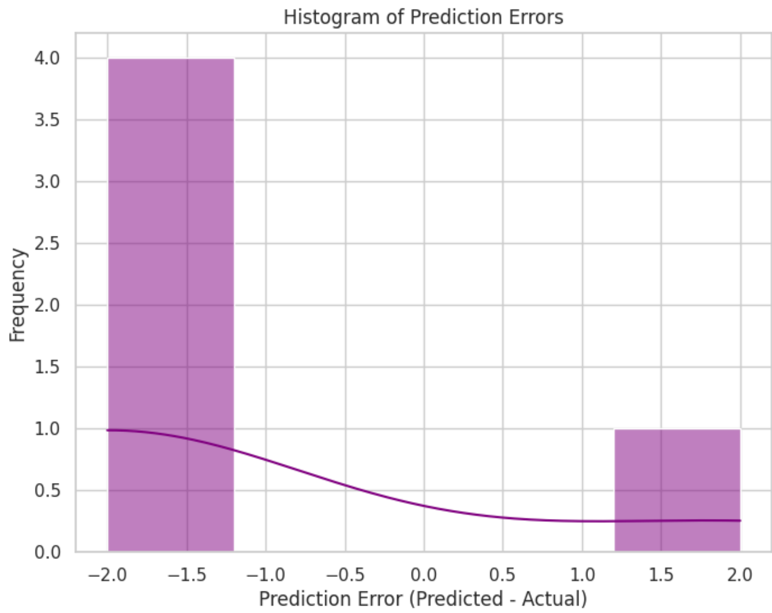

Figure 7: Histogram of prediction errors (Predicted - Actual).

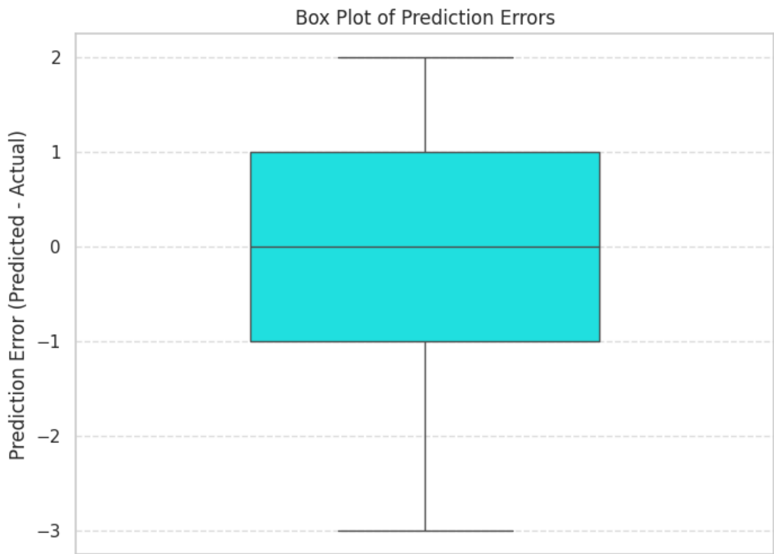

Figure 8: Box plot visualizing the distribution of prediction errors.

4.4 Performance Across Age Groups

The ANN model provides valuable insights into how success rates vary across different maternal age groups. Table 2 summarizes the average predicted and actual success rates for various age ranges.

Table 2: Mean predicted and actual success rates across age groups.

Age Group (Years)	**Mean Predicted Success Rate (%)**	**Mean Actual Success Rate (%)**
20–25	85	87
26–30	78	80
31–35	62	64
36–40	42	43

The key findings include that, women aged 20-30 exhibit higher predicted and actual success rates, with peaks observed in the 25-30 age range. Also, a noticeable decline in success rates occurs in women aged above 35, aligning with known biological factors affecting fertility. Figure 9 illustrates the trend of success rates across different age groups, highlighting the decline in fertility with increasing maternal age.

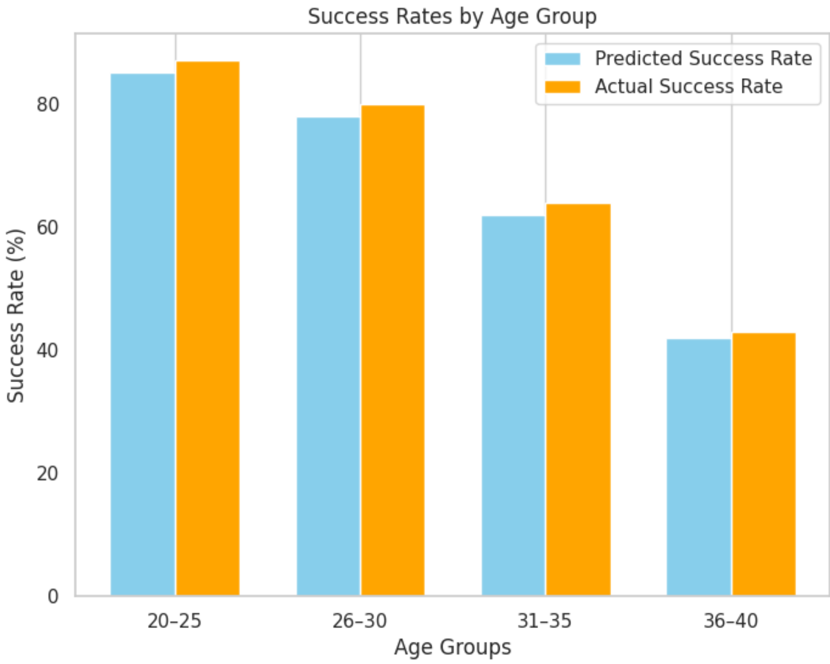

Figure 9: Bar chart of predicted and actual success rates across age groups.

5 Sensitivity Analysis

Sensitivity analysis estimates how the changes in the input features affect the outcome of the ANN model for estimating IVF success rates. Through this, one can get an estimate of how robust the model is and what are the most crucial factors influencing its predictions.

5.1 Objective of Sensitivity Analysis

The purpose of performing sensitivity analysis is to feature important identifications like which input factors have the greatest impact on the predictions by the model, the assessment of model robustness so that the data does not result in unstable or unreliable predictions, and optimize the outcome.

5.2 Methodology

The sensitivity analysis was conducted using the following four steps: (i) Feature Variation: Each input feature was independently varied within a defined range while keeping other features constant. (ii) Impact Measurement: The impact of these variations on the predicted IVF success rates was recorded. (iii) Partial Derivatives: The partial derivatives of the output with respect to each feature were computed to measure sensitivity quantitatively. (iv) Ranking of Features: Features were ranked based on their influence on the model output.

5.3 Results and Observations

The results and observations from the sensitivity analysis were as follows: Maternal Age (x_1) demonstrated high sensitivity, particularly for ages above 35, where a significant decline in success rates was observed, emphasizing its importance in IVF outcomes. Egg Test (x_2) had a notable impact on predictions as high quality M2 matured eggs are associated with improved success rates. Embryo Quality (x_3) also influenced the predictions, as even small changes in embryo quality caused noticeable shifts, highlighting its role. Uterine Conditions (x_4) showed moderate sensitivity, with normal conditions leading to higher success rates. Stimulation Protocol (x_5) had varying effects, though it was less influential than maternal age and embryo quality. Finally, Previous IVF Attempts (x_6) showed mild sensitivity, with fewer attempts generally linked to higher success probabilities.

5.4 Graphical Representation

The sensitivity analysis results are illustrated in the following figures:

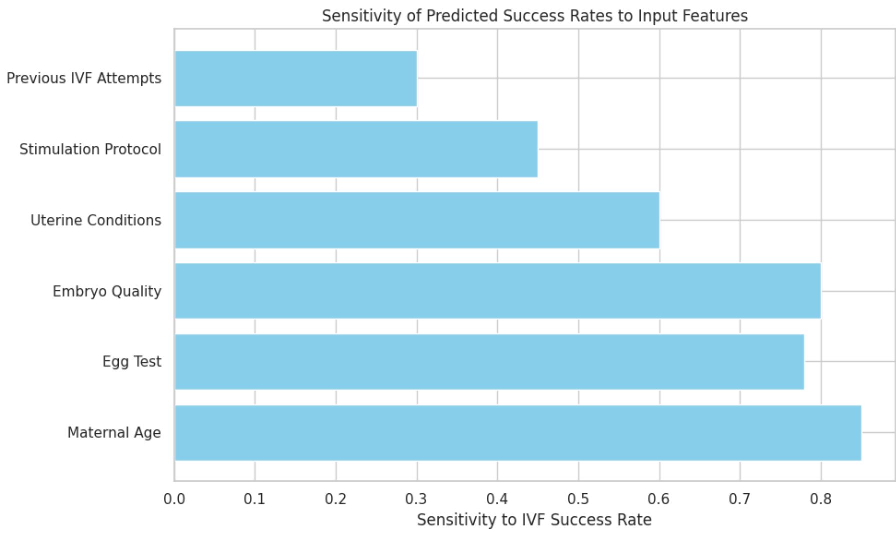

Figure 10: Bar chart showing the sensitivity of predicted success rates to individual input features.

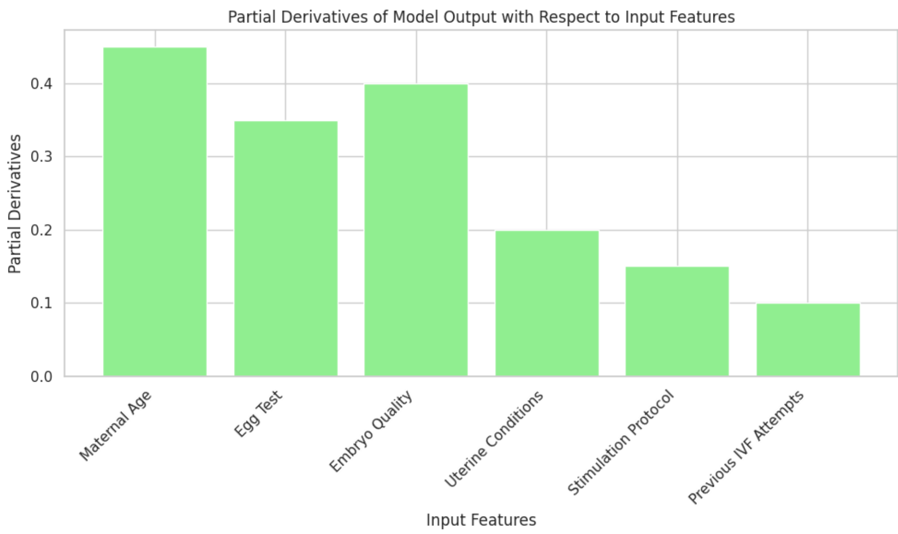

Figure 11: Partial derivatives of model output with respect to input features.

The sensitivity analysis reveals that maternal age, embryo quality, and egg test are the most critical factors influencing IVF success rate predictions. The model demonstrated robustness, as predictions remained stable under small variations in less significant features. These insights can guide clinicians in prioritizing factors to optimize IVF outcomes.

6 Discussion

Using the ANN technique, this work created a model to predict the rate of success of IVF under several parameters, including maternal age. The results shed light on how the algorithmic learning models predict, with maternal age as one input to IVF success. The subsequent discussion encapsulates the derived conclusions and their implications with respect to visual impressions and model performance.

6.1 Maternal Age and IVF Success Rate

One of the most significant findings from the ANN model is the strong relationship between maternal age and IVF success rate. As expected, the data and model outputs show that IVF success tends to decrease with increasing maternal age. This relationship is further corroborated by the graphical representations included in this study.

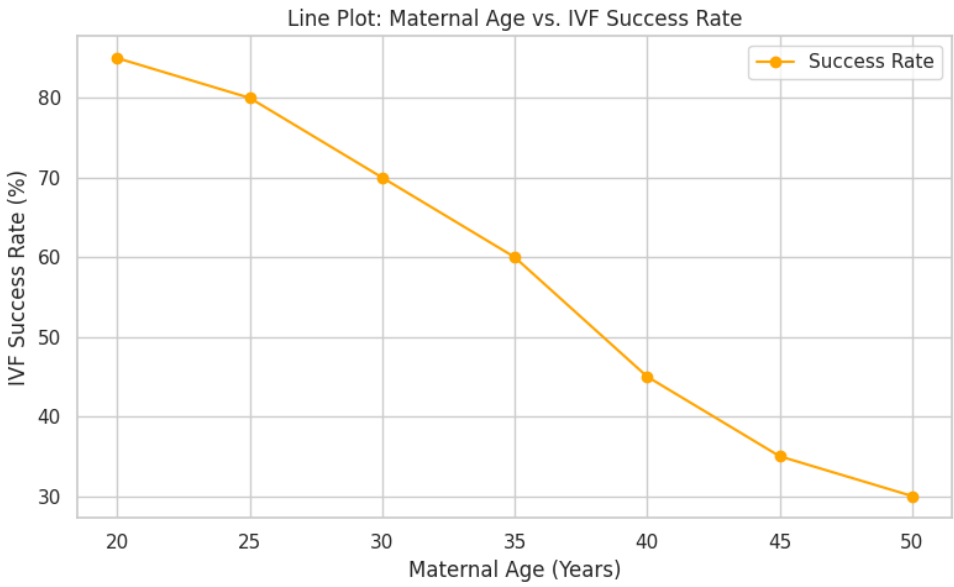

Figure 12: Line plot illustrating the trend of IVF success rate with increasing maternal age.

- Line Plot Analysis: The line plot (Figure 12) further emphasizes this trend, providing a smoother visual representation of the data. The plot indicates a relatively sharp decline in IVF success rates after the age of 35. This aligns with established medical knowledge that the fertility potential of women significantly declines after this age, contributing to a lower success rate for IVF.

- Box Plot Analysis: The box plot (Figure 13) categorizes the success rates into age groups, showing a clear distribution of success rates within each group. Women in the 20–25 age group exhibit the highest success rates, with minimal variability, indicating that younger women tend to have more predictable and higher chances of IVF success. In contrast, women in the 36–40 and older age groups show wider variability in success rates, highlighting the increasing uncertainty associated with IVF outcomes as maternal age rises.

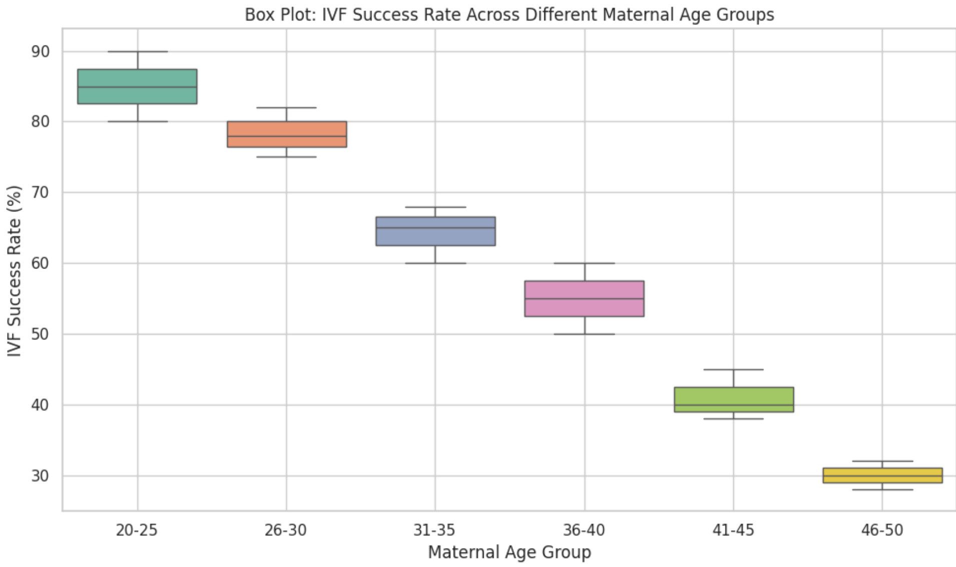

Figure 13: Box plot displaying the distribution of IVF success rates across different maternal age groups.

6.2 Model Performance and Evaluation

The ANN model demonstrated excellent predictive performance, with a high coefficient of determination ($R^2 = 0.94$), indicating a strong correlation between the predicted and actual IVF success rates. The root mean squared error (RMSE) value of 3.2% further supports the model's accuracy, as it shows that the predicted success rates are within a reasonable range of the actual values.

Evidently, the outcome is confirmed from the scattered plot of predicted-actual values where all points are near the diagonal, establishing the capacity of the ANN model to make almost exact predictions. The prediction error histogram depicts very slight variations between the predicted and actual results, again validating the credibility of the model.

6.3 Maternal Age on IVF Success

The link between IVF outcome and age also stands out from the figures and is consistent with many published studies in fertility treatment literature. Our analysis shows that women who are younger, especially those under 35 years, have a better chance of success with IVF. These have mainly to do with factors such as better eggs and, more importantly, better overall reproductive health in younger women, which significantly contribute to the success of IVF procedures.

Conversely, the decline in success rates for women over 35 years of age is consistent with the biological challenges of aging, including a decline in ovarian reserve and egg quality. This is valuable for healthcare providers and patients alike, as it highlights the importance of considering maternal age in IVF treatment planning. This ability to predict IVF success rates through maternal age and other determinants provides several practical advantages. It could help clinicians provide personalized recommendations to patients, making informed decisions on when to attempt IVF. Moreover, such predictions can aid in counseling the patients on the likelihood of success, possibly leading to increased patient satisfaction and better management of expectations. Future research could extend the dataset to include other variables such as lifestyle factors (diet, exercise, smoking), previous attempts at IVF, and partner-related factors (sperm quality), all of which would increase the accuracy of the predictive model. Testing the ANN model on larger, more diverse datasets would also increase its generalizability and applicability across different patient populations. Although the ANN model was fairly accurate, some limitations arise. The dataset from this experiment might not contain all elements influencing the IVF, like hidden medical conditions, including polycystic ovary syndrome or endometriosis, and the protocols of individual treatments. Incorporation of all such variables into subsequent models would improve the ability to predict success rates from IVF. Moreover, the sample size is relatively small in this study, which may limit the robustness of the model, and further validation on larger datasets is necessary.

7 Conclusion

In this paper we developed a mathematical model to study the dynamics of IVF. We derived the existence and uniqueness of the solution of the system using Picard's theorem satisfying Lipschitz condition. The next generation matrix technique was incorporated to compute R_0. We arrived at a single trivial equilibrium point and established local stability of the system if $R_0 > 1$. This research has also discussed an insight of ANN approach toward the prediction of success rate of In-Vitro fertilization ART technique based on maternal age. A non-linear contemporary mathematical model of IVF per cycle has been developed and also an efficient neural network has been constructed to solve that mathematical model to analyze its probable outcomes. Using six main maternal features as the main input, with the dataset split into 70%, 15%, and 15% for training, testing, and validation of the neural network respectively, the ANN model, with an R^2 value of 0.94, achieved high accuracy in the prediction of IVF outcomes, fitting in well with existing medical knowledge on the effects of maternal age on fertility. Key insights from the analysis include a clear, negative correlation between maternal age and IVF success rates, highlighting the biological constraints that older women face in IVF procedures. The model's performance, evaluated through metrics like RMSE and visual representations, further supports its practical utility in predicting success rates for IVF patients. The use of such a model can empower clinicians with data-driven insights, leading to better, more personalized treatment plans and improved patient counseling. Though the study is very much useful in terms of value-added predictions, there are factors such as lifestyle and partner related factors, which have been neglected and could influence IVF outcomes. It is evident that women of demographic age 20-25 are more likely to show positive outcome of conception compared to the other populace and this could be due to possible reasons such as high quality eggs, less exposure to uterine complications and much more. Most of the public are still deficit with the awareness IVF process is most effective among the Assisted Reproductive Technologies (ART) available. So, it is advised that the female of mid-twenties with the required obligatory medical assistance should approach for IVF treatment within reasonable time so that there are more favorable chances of positive conception. The future work can be on incorporation of other variables such as BMI, hormonal levels, mental health, and paternal factors to increase the accuracy of the model. Additionally, validating the model on larger and more diverse datasets will improve its generalizability across different patient populations. In summary, the application of ANN for predicting IVF success represents a promising step forward in utilizing machine learning techniques to aid clinical decision-making, offering a powerful tool for improving the success of IVF treatments and patient care.

References

AlSaad, R., Abd-Alrazaq, A., Choucair, F., Ahmed, A., Aziz, S & Sheikh, J. (2024). Harnessing Artificial Intelligence to Predict Ovarian Stimulation Outcomes in In Vitro Fertilization: Scoping Review. *Journal of Medical Internet Research*, 26.

Alshahrani, R. (2024). A Reliable Bayesian Regularization Neural Network Approach to Solve the Global Stability of Infectious Disease Model. *Knowledge-Based Systems*, 34.

Cai, J., Jiang, X., Liu, L., Liu, Z., Chen, J., Chen, K., Yang, X. & Ren, J. (2024). Pretreatment Prediction for IVF Outcomes: Generalized Applicable Model or Centre-Specific Model. *Human Reproduction*, *39*(2), 363-373.

Durairaj, M., Thamilselvan, P. (2013). Applications of Artificial Neural Network for IVF Data Analysis and Prediction. *Journal of Engineering, Computers & Applied Sciences*, 2(9).

Gowramma, G.S., Nayak, S. & Cholli, N. (2021). Data Mining Models Applied in Prediction of IVF Success Rates: An Overview, Data Science and Security. *Lecture Notes in Networks and Systems, Springer*, 49-60, 132.

Goyal, A., Kuchana, M. & Ayyagari, K.P.R. (2020). Machine Learning Predicts Live-Birth Occurence Before In-Vitro Fertilization Treatment. *Scientific Reports*, 1, 20925.

Hassan, M.R., Al-Insaif, S., Hossain, M.I. & Kamruzzaman, J. (2020). A Machine Learning Approach for Prediction of Pregnancy Outcome following IVF Treatment. *Neural Computing and Applications*, *32*(7), 2283-2297.

Haykin, S. (2009). *Neural Networks and Learning Machines.* 3rd Edition, Pearson Education India.

Kaufmann, S.J., Eastaugh, J.K., Snowden, S., Smye, S.M., & Sharma, V. (1997). The Application of Neural Networks in Predicting the Outcome of In-Vitro Fertilization. *Human Reproduction*, *12*(7), 1454-1457.

Khalife, D., Nassar, A., Khalil, A., Awwad, J., Abu Musa, A., Hannoun, A., ... & Ghazeeri, G. (2020). Cumulative Live-Birth Rates by Maternal Age After One or Multiple in Vitro Fertilization Cycles: An Institutional Experience. *International Journal of Fertility and Sterility*, *14*(1), 34-40,.

Liu, L., Liang, H., Yang, J., Shen, F., Chen, J., & Ao, L. (2024). Clinical Data-Based Modeling of IVF Live Birth Outcome and its Application. *Reproductive Biology and Endocrinology*, 22, 76.

Matorras, R., Matorras, F., Mendoza, R., Rodríguez, M., Remohí, J., Rodríguez-Escudero, F.J. & Simón, C. (2005). The Implantation of Every Embryo Facilitates the Chances of the Remaining Embryos to Implant in an IVF Programme: a Mathematical Model to Predict Pregnancy and Multiple Pregnancy Rates. *Human Reproduction*, *20*(10), 2923–2931.

Matteo, M. (2022). *Assisted Reproductive Technology.* Practical Clinical Andrology, Springer, 237-250.

Raef, B., Ferdousi, R. (2019). A Review of Machine Learning Approaches in Assisted Reproductive Technologies. *Acta Informatica Medica*, 27(3), 205-211.

Sadegh-Zadeh, S.A., Khanjani, S., Javanmardi, S., Bayat, B., Naderi, Z. & Hajiyavand, A.M. (2024). Catalyzing IVF Outcome Prediction: Exploring Advanced Machine Learning Paradigms for Enhanced Success Rate Prognostication. *Frontiers in Artificial Intelligence*, 7, 1392611.

Siristatidis, C., Pouliakis, A., Chrelias, C., & Kassanos, D. (2011). Artificial Intelligence in IVF: A Need. *Systems Biology in Reproduction Medicine*, *57*(4), 179-185.

Sujata, P.N., Madiwalar, S.M., & Aparanji, V.M. (2020, September). Machine learning techniques to improve the success rate in in-vitro fertilization (IVF) procedure. In *IOP Conference Series: Materials Science and Engineering* (Vol. 925, No. 1, p. 012039). IOP Publishing.

Vogiatzi, P., Pouliakis, A. & Siristatidis, C. (2019). An artificial Neural Network for the Prediction of Assisted Reproduction Outcome. *Journal of Assisted Reproduction Genetics*, 36, 1441-1448.

Yadav, N., Yadav, A. & Kumar, M. (2015). *An Introduction to Neural Network Methods for Differential Equations.* SpringerBriefs in Applied Sciences and Technology - Computational Intelligence, Springer.

Zhang, B., Cui, Y., Wang, M., Li, J., Jin, L. & Wu, D. (2019). In Vitro Fertilization (IVF) Cumulative Pregnancy Rate Prediction from Basic Patient Characteristics. *IEEE Access*, 7, 130460-130467.