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# MATHEMATICAL MODEL OF THE DURABILITY OF MATERIALS USED IN THE MANUFACTURE OF NANOSATELLITES

**Mehman Hasanov**[1,2*], **Sahib Piriev**[1], **Khagani Abdullayev**[2], **Emin Payizov**[1], **Turana Rasullu**[1], **Nurana Camalzada**[1], **Cavid Namazov**[1]

[1]Azerbaijan Technical University, Baku, Azerbaijan
[3]National Aviation Academy, Baku, Azerbaijan

**Abstract.** This paper presents mathematical models for predicting the durability of materials intended for use in the manufacturing of nanosatellites, considering both simple and complex stress states. The material damage process is examined in two phases. In the first phase, analytical formulas for the incubation period (latent damage) are derived using a criterion-based approach, considering fixed, exponential, and Abel kernels for the damage operator. In the second phase, an integral equation of the Volterra type, characterizing the failure process, is derived and solved based on the classical strength criterion, accounting for hereditary-type damaged materials. Time dependence graphs of the damage process were constructed using the obtained results and comparisons were made. The research determined that for the exponential case of the damage operator kernel, the incubation period constitutes 12.5% and 5.6% of the total failure time for mechanical memory coefficients of 0.3 and 0.2, respectively.

**Keywords**: Damage, long-term strength, damage function, damage parameter.
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[*]**Corresponding author:** Mehman Hasanov, Department of Radiotechnics and Telecommunication, Azerbaijan Technical University, Baku, Azerbaijan, Tel.: +994 70 211 22 83, e-mail: *mehman.hasanov@aztu.edu.az*
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## 1 Introduction

Various practical problems arise during the design and operation of nanosatellite structures. One of the main challenges with nanosatellites, which are equipped with complex structural systems and various components, is that the long-term strength conditions of such structures are often not considered in design calculations. This leads to sudden premature failures during their operation. Therefore, the use of new types of composite and polymer materials in the structural elements, especially when operating under special or unusual conditions, requires the calculation of their durability. Long-term strength is associated with the gradual damage process that occurs in materials over time. As a result, studying the damage process in operational equipment parts is considered one of the most pressing problems of modern times. The longevity of nanosatellites directly depends on the nature of the damage process in their materials. In this sense, the study of the damage process becomes essential. The damage process is a common issue for the mechanics of continuous media, solid body mechanics, and materials science. Damage mechanics evolves in two directions: 1) constructing models that describe the material damage process based on general theoretical considerations and experimental data; 2) solving relevant problems using these models (Hasanov et al., 2024a,b; Lokoshenko et al., 2018).

The structure of the material in newly developed devices significantly influences the damage process, and for this reason, the damage can be complex and unstable. Moreover, the damage process is also strongly dependent on external factors. This process depends significantly on external influences, such as the nature of the load, the thermal regime, surface effects, the turbulence parameters of the operating environment, and so on. All of these factors affect the stress state of the nanosatellite's material, which in turn leads to the damage of material. One type of failure is associated with random defects that form and accumulate in materials, which is described by the term "breakdown." This type of damage process is known as dispersed damage. If the stress state of the material is homogeneous (for example, if any rod-shaped element of the nanosatellite is being stretched), the damage process increases uniformly throughout the volume. However, in a non-homogeneous stress state, the damage process must be analyzed in two stages: latent damage (the incubation period) and the open stage of the damage process (Lokoshenko et al., 2018).

Microcracks and other defects accumulate during the latent damage stage (within a certain time interval $0 \leq t < t^*$), and at $t = t^*$ localized damage begins to occur. These scattered microcracks and other defects combine around these localized damaged areas to form larger microcracks. For example, experiments conducted on nanosatellite materials related to fatigue show that, damages continuously accumulate and disperse in the initial stage, and microcracks begin to intensively develop at the end of this stage. Experiments on materials show that during fatigue damages, the incubation period constitutes 80-90% of the total damage time. Once the latent damage process transitions to the observable open stage of damage, the speed of the damage process increases (Lokoshenko et al., 2018).

There are two approaches used to study damage problems. The first is the criterion-based approach. This approach is based on creating a criterion that defines the damage process over the operational lifespan of materials. To create such a criterion, an "equivalent" stress is used. An analytical review of research using the criterion-based approach is provided in (Lokoshenko et al., 2018; Lokoshenko, 2012). This approach is mainly used in problems where the strength of components in the stress tensor, ühich is independent of time is examined.

The second approach is the kinetic approach. Numerous studies conducted on various materials have suggested the "kinetic method" approach for investigating long-term strength issues (Lokoshenko, 2012; Kachanov, 2013). The core of this approach is the introduction of a scalar function called the "damage parameter" $\omega(t)$, which characterizes the state of the material at any arbitrary time t during operation. The value of $\omega(0) = 0$ corresponds to the initial state of the material (before operation), and the value $\omega(t^*) = 1$ corresponds to the complete damage state of the material (Lokoshchenko, 2016a).

By using the definition of the scalar damage function $\omega(t)$ for a uniaxial stress state, the damage parameter can be considered as the relative reduction of the cross-sectional area of a cylindrical rod under tensile stress. The obtained results can be applied to various parts of the nanosatellite. Then, at time $t = 0$, the normal stress in the cross-section will be as follows:

$$\sigma_0 = \frac{F}{A_0}. \tag{1}$$

Here $A_0 = \pi r^2$ is the cross-sectional area of the rod. After loading, the area begins to decrease as a result of the growth of the linear dimensions of the defects formed in the cross section of the bar.

$$A(t) = A_0 - \Delta A(t). \tag{2}$$

Here, the time function $\Delta A(t)$ is the function that expresses the reduction of the area depending on the time, $F$ is the drag force. Then the normal stress in the section will also vary with time:

$$\tilde{\sigma}(t) = \frac{F}{A(t)}. \tag{3}$$

If (2) is taken into account in clause (3),

$$\tilde{\sigma}(t) = \frac{F}{A_0 - \Delta A(t)} = \frac{F}{A_0 \left(1 - \frac{\Delta A(t)}{A_0}\right)}. \tag{4}$$

Here

$$\omega(t) = \frac{\Delta A(t)}{A_0}. \tag{5}$$

This function is a scalar function that characterizes the damage process. If (5) is taken into account in formula (4), the law of change of normal stress depending on time will be as follows:

$$\tilde{\sigma}(t) = \frac{F}{A_0 \left(1 - \omega(t)\right)}. \tag{6}$$

If (1) is taken into account in formula (6),

$$\tilde{\sigma}(t) = \frac{\sigma_0}{1 - \omega(t)}. \tag{7}$$

Using this approach, many theories of long-term material damage have been developed based on experiments conducted by various researchers. This has accelerated the development of damage mechanics. However, alongside the advantages of these theories, there are also some limitations. The achievements and shortcomings of damage mechanics have been analyzed in (Vilchevskaya et al., 2015).

In the research work (Xiao et al., 2019), the authors begin with fundamental studies and provide an analysis of material creep and long-term strength. Creep and long-term strength issues are analyzed from a theoretical (phenomenological or structural) perspective. The study pays detailed attention to the structural mechanisms of creep, including factors like pore sizes in materials, diffusion processes, and so on.

It is clear that all theoretical research is based on experiments. Such experiments have been conducted on materials loaded under various conditions. In (Lu et al., 2019), the damage process of pipe-shaped metal samples under creep conditions at high temperatures was studied. In this work, the long-term damage of pipes subjected to both tensile forces and internal pressure was also investigated.

In studies (Yao et al., 2007; Yuan et al., 2015; Wu, 2019), long-term strength problems under internal pressure were investigated using diffusion equations to account for the concentration distribution of the aggressive environment. Based on the results obtained, the effect of the aggressive environment on the damage process was determined.

There are many generalized kinetic models for calculating the structural degradation of materials. The use of such models is mainly convenient for calculating the durability of classical materials under simple stress conditions. Since the constants expressing the mechanical properties of materials (e.g., Poisson's ratio, instantaneous strength limit) are not included in these models, their application to the calculation of the durability of materials under plane or complex stress states is not sufficiently effective (errors are significant).

For damageable materials, the development of differential equations that characterize the damage process under both simple and complex stress conditions, while incorporating the mechanical properties of the materials into these equations to enhance their accuracy, is considered a pressing issue.

# 2 Mathematical Modeling

## 2.1 Scalar Parameter of Damage

In the initial case, it is appropriate to construct a mathematical model for long-term damage in materials under a homogeneous stress state. For this, consider a cylindrical rod with a radius $r$ subjected to a tensile force $F$. The damage process is examined in two stages for model construction. In the first stage, microcracks and other defects will form in the material under the influence of the force $F$ at the initial moment of loading. The damage process that occurs within the time interval $0 \leq \tau \leq t_0$ is called the latent damage stage. During this phase, the material retains its functional capability. In the second stage, within the time interval $t_0 \leq \tau \leq t^*$, the microcracks and other defects merge to form macrocracks, which increase uniformly throughout the volume of the material. Complete failure occurs at $\tau = t^*$, and the material fully loses its functionality at this moment. Under these conditions, for the damage function $\omega(t)$, we can accept $\omega(t_0) = 0$ and $\omega(t^*) = 1$.

Using equation (7), the relationship between stress and deformation under uniaxial stress can be expressed as follows:

$$\tilde{\varepsilon} = \frac{\tilde{\sigma}}{E\,(1-\omega(t))}. \tag{8}$$

The relationship between stress and strain for materials with hereditary type damage is expressed as follows (Yuan et al., 2015).

$$\tilde{\varepsilon} = \frac{1}{E}\left(\tilde{\sigma}(t) + \int_0^t B(\tau)\tilde{\sigma}(\tau)\,d\tau\right). \tag{9}$$

The classic robustness condition:

$$\tilde{\varepsilon} \leq [\varepsilon]. \tag{10}$$

Here, $[\varepsilon]$ is the strain strength limit. Then, the strain strength limit can be expressed as the stress strength limit as follows:

$$[\varepsilon] = \frac{[\sigma]}{E\,(1-\omega)}. \tag{11}$$

After writing (9) to (7), if we write (9) and (11) in (10), the Volterra-type integral equation is obtained as follows:

$$\frac{\sigma_0}{1-\omega(t)} + \sigma_0 \int_0^t \frac{B(\tau)}{1-\omega(\tau)}\,d\tau = \frac{[\sigma]}{1-\omega(t)}. \tag{12}$$

or

$$(1+B^*)\tilde{\sigma}(\tau) = \frac{[\sigma]}{1-\omega(t)}. \tag{13}$$

Here, $B^*$ is an integral type operator characterizing the damage process, and $B(\tau)$ is the kernel of this operator.

$$B^* \cdot 1 = \int_0^t B(\tau)\,d\tau. \tag{14}$$

The first stage of the dissolution process (incubation period) is considered. For this, it is assumed that there is no defect in the stick before loading. The strength criterion for maximum stress is written as follows by using (13) (Wu, 2019; Wu et al., 2024; Grydzhuk et al., 2022):

$$(1+B^*)\sigma_{\max} = [\sigma]. \tag{15}$$

In the case of simple tension, the maximum stress is expressed by the normal stress occurring in the cross section of the bar. Therefore, the first breakdown will occur due to this stress.

$$\sigma_{\max} = \frac{F}{A_0}. \tag{16}$$

If this formula (15) is expressed in the strength criterion, the following equation is obtained to determine the initial decay time:

$$\int_0^{t_0} B(\tau)\, d\tau = g - 1. \tag{17}$$

Here, $g = \frac{[\sigma]}{\sigma_0}$, where $[\sigma]$ is the instantaneous strength limit of the flawless material.

The incubation period for the three forms of the $B(\tau)$ kernel included in equation (17) is found as follows (Piriev et al., 2022; Akhmedov et al., 2024):

$$B(t) = 1; \qquad t_0 = g - 1, \tag{18}$$

$$B(t) = e^{-\alpha t}; \qquad t_0 = \frac{1}{\alpha} \ln\left[1 + \alpha(1 - g)\right]^{-1}, \tag{19}$$

$$B(t) = t^{-\alpha}; \qquad t_0 = \left[(1 - \alpha)(g - 1)\right]^{\frac{1}{1-\alpha}}. \tag{20}$$

It follows from equations (18)-(20) that determine the initial damage time that, in the case of a singular or constant kernel, damage will occur even at the smallest value of the tensile force, and as this force decreases, the value of $t_0$ will increase. In the case of a regular exponential kernel, the lower boundary of the damage time is obtained.

To examine the second phase of the damage stage, we can rewrite equation (13) as follows (Grydzhuk et al., 2022):

$$\int_0^t B(t, \tau) \frac{1}{1 - \omega(\tau)}\, d\tau = \frac{g - 1}{1 - \omega(t)}. \tag{21}$$

If $B(t, \tau) = B_0 = \text{const}$, the integral equation was solved by the differentiation method. For this, both sides of equation (21) are differentiated with respect to $t$:

$$\frac{d\omega}{dt} = \frac{B_0(1 - \omega)}{g - 1}. \tag{22}$$

Solving the resulting differential equation (22) with the initial condition $\omega(t = t_0) = 0$, the damage parameter is determined as follows:

$$\omega(t) = 1 - e^{\frac{B_0}{g-1} \cdot (t - t_0)}. \tag{23}$$

By the same rule, the damage parameter is determined by substituting $B(t, \tau) = e^{-\alpha(t-\tau)}$ in equation (12). For this, first, equation (12) is expressed as follows:

$$\int_0^t e^{-\alpha(t-\tau)} \cdot \frac{1}{1 - \omega(\tau)}\, d\tau = \frac{g - 1}{1 - \omega(t)}. \tag{24}$$

Then the mass integral in this equation is determined:

$$\int_0^t e^{\alpha\tau} \frac{1}{1 - \omega(\tau)}\, d\tau = \frac{g - 1}{1 - \omega(t)} \cdot e^{\alpha t}. \tag{25}$$

After differentiating equation (25), the next integro-differential equation is obtained:

$$\int_0^t \frac{\partial e^{-\alpha(t-\tau)}}{\partial t} \cdot \frac{1}{1-\omega(\tau)}\, d\tau + \frac{1}{1-\omega(t)} = \frac{g-1}{(1-\omega(t))^2} \frac{d\omega(t)}{dt}. \tag{26}$$

or

$$-\alpha e^{-\alpha t} \int_0^t e^{\alpha\tau} \frac{1}{1-\omega(\tau)}\, d\tau + \frac{1}{1-\omega(t)} = \frac{g-1}{(1-\omega(t))^2} \frac{d\omega(t)}{dt}. \tag{27}$$

By writing equation (27) in (25), the following differential equation is obtained:

$$\frac{d\omega}{dt} = \frac{1-\alpha(g-1)}{g-1} \cdot (1-\omega). \tag{28}$$

The analysis of equation (28) shows that in the case of a constant kernel, the rate of change of the damage parameter over time depends only on the parameter $g$. Thus, as the value of $g$ increases, the rate of material damage decreases. In other words, the material becomes more resistant to elongation. It is also evident from equation (18) that as the value of the $g$ parameter increases, the incubation period lengthens. When $g-1=0$ in equations (22) and (28), $\frac{d\omega}{dt} = \infty$, which means that the effective force equals the instantaneous strength limit of the material. Since the function $\omega(t)$ is monotonically increasing, the right-hand side of equation (28) must be positive. That is, the condition $\alpha(g-1)(1-\omega) < 1$ must be satisfied. Under real conditions, the rate of increase is not zero. Failure occurs when the value of the damage parameter reaches a critical value, $\omega = \omega_* < 1$.

In the case of an exponential kernel (when the material has relative fluidity), one of the factors affecting the material's durability is the value of the $\alpha$ parameter.

## 2.2 Vectorial Parameter of Damage

As can be seen, the scalar damage parameter is only suitable for simple stress conditions. It is clear that the defects (voids, micropores, microcracks) that accumulate in the damage process are determined by the loads that cause them. As is known, microcracks typically develop in a direction perpendicular to the maximum principal stress. The growth of these microcracks leads to the destruction of intergranular bonds in polycrystals, which ultimately results in damages.

To describe such types of damage, the scalar damage parameter is insufficient. Therefore, in such cases, to accurately express the damage of materials, vector or tensor concepts of damage must be used. A comprehensive review of research using this approach is provided in (Akhmedov et al., 2024).

In a complex stress state, the rate of defect accumulation $\dot{\omega}$ depends on the normal stress acting on each plane. In such a situation, when the damage parameter $\omega$ reaches its limit value in any direction, localized damage occurs, and after the damage front spreads through the object, complete damage happens (Piriev, 2023).

For a complex stress state, the vectorial damage parameter is defined as (Piriev et al., 2022):

$$\varpi = \sqrt{\omega_1^2 + \omega_2^2 + \omega_3^2}. \tag{29}$$

Here, the parameters $\omega_i$ are related to the principal stresses $\sigma_i$ where $i = 1, 2, 3$ as follows:

$$\frac{d\omega_i}{dt} = \begin{cases} f(\sigma_i, \omega_i), & \sigma_i > 0 \\ 0, & \sigma_i \le 0 \end{cases}. \tag{30}$$

These relationships describe the projections of the damage vector in the direction of the principal stresses during the creep process. The damage vector must satisfy the conditions $\varpi(0) = 0$ and $\varpi(t^*) = 1$.

Using the concept of the vectorial damage parameter, let's consider the long-term damage of a structure made of hereditary-type material under a complex stress state. Suppose there is a prismatic body in a complex stress state. Let the surface areas of the body be $A_1, A_2, A_3$, and the normal stresses acting on each surface be $\sigma_1, \sigma_2, \sigma_3$ (with $\sigma_1 \geq \sigma_2 \geq \sigma_3$). Then, using relationships from equation (7), the law of variation of the normal stress acting on each surface can be written as follows:

$$\tilde{\sigma}_1(t) = \frac{\sigma_1}{1-\omega_1(t)}; \qquad \tilde{\sigma}_2(t) = \frac{\sigma_2}{1-\omega_2(t)}; \qquad \tilde{\sigma}_3(t) = \frac{\sigma_3}{1-\omega_3(t)}. \tag{31}$$

The operator form of the material constants included in the system of equations (32) will be as follows (Grydzhuk et al., 2022; Piriev et al., 2022):

$$\frac{1}{E^*} = \frac{1}{E}(1+B^*); \qquad \frac{\nu^*}{E^*} = \frac{\nu}{E}(1+B^*). \tag{32}$$

Here, $E$ and $\nu$ are the modulus of elasticity and Poisson's ratio of the material, respectively.

After writing expression (32) in (33), using the robustness conditions (10)-(11), the robustness criteria for each direction are written as follows:

$$\begin{cases} (1+B^*)\cdot[\tilde{\sigma}_1 - \nu(\tilde{\sigma}_2+\tilde{\sigma}_3)] = \frac{[\sigma]}{1-\omega_1(t)}, \\ (1+B^*)\cdot[\tilde{\sigma}_2 - \nu(\tilde{\sigma}_1+\tilde{\sigma}_3)] = \frac{[\sigma]}{1-\omega_2(t)}, \\ (1+B^*)\cdot[\tilde{\sigma}_3 - \nu(\tilde{\sigma}_1+\tilde{\sigma}_2)] = \frac{[\sigma]}{1-\omega_3(t)}. \end{cases} . \tag{33}$$

If we write (15) and (31) in (33) and differentiate each equation included in the system with equation (33) with respect to $t$ for the case that the kernel of the creep operator is constant, we get the following system of equations including the projections of the damage vector:

$$\begin{cases} \frac{1-g_1^{(1)}}{(1-\omega_1)^2}\frac{d\omega_1}{dt} - \nu\left(\frac{g_1^{(2)}}{(1-\omega_2)^2}\frac{d\omega_2}{dt} + \frac{g_1^{(3)}}{(1-\omega_3)^2}\frac{d\omega_3}{dt}\right) = \nu\left(\frac{g_1^{(2)}}{1-\omega_2} + \frac{g_1^{(3)}}{1-\omega_3}\right) - \frac{1}{1-\omega_1}, \\ \frac{1-g_2^{(2)}}{(1-\omega_2)^2}\frac{d\omega_2}{dt} - \nu\left(\frac{g_2^{(1)}}{(1-\omega_1)^2}\frac{d\omega_1}{dt} + \frac{g_2^{(3)}}{(1-\omega_3)^2}\frac{d\omega_3}{dt}\right) = \nu\left(\frac{g_2^{(1)}}{1-\omega_1} + \frac{g_2^{(3)}}{1-\omega_3}\right) - \frac{1}{1-\omega_2}, \\ \frac{1-g_3^{(3)}}{(1-\omega_3)^2}\frac{d\omega_3}{dt} - \nu\left(\frac{g_3^{(1)}}{(1-\omega_1)^2}\frac{d\omega_1}{dt} + \frac{g_3^{(2)}}{(1-\omega_2)^2}\frac{d\omega_2}{dt}\right) = \nu\left(\frac{g_3^{(1)}}{1-\omega_1} + \frac{g_3^{(2)}}{1-\omega_2}\right) - \frac{1}{1-\omega_3}. \end{cases} \tag{34}$$

Here,

$$g_1^{(1)} = \frac{[\sigma]}{\sigma_1}; \quad g_1^{(2)} = \frac{\sigma_2}{\sigma_1}; \quad g_1^{(3)} = \frac{\sigma_3}{\sigma_1};$$

$$g_2^{(2)} = \frac{[\sigma]}{\sigma_2}; \quad g_2^{(1)} = \frac{\sigma_1}{\sigma_2}; \quad g_2^{(3)} = \frac{\sigma_3}{\sigma_2};$$

$$g_3^{(3)} = \frac{[\sigma]}{\sigma_3}; \quad g_3^{(1)} = \frac{\sigma_1}{\sigma_3}; \quad g_3^{(2)} = \frac{\sigma_2}{\sigma_3};$$

For a complex stress state, the system of equations (34) can be somewhat simplified to qualitatively analyze the damage process. Let us assume that the prismatic body is in the form of a cube, and the normal stresses acting on its faces are equal.

$$A_1 = A_2 = A_3 = A; \quad \sigma_1 = \sigma_2 = \sigma_3 = \sigma.$$

Then, the system of equations (34) will be expressed by an equation:

$$\varepsilon = \frac{1}{E^*}\tilde{\sigma} - \frac{2\nu^*}{E^*}\tilde{\sigma},$$

or

$$\int_0^t B(t,\tau)\frac{1}{1-\omega(\tau)}\,d\tau = \frac{g-1+2\nu}{(1-\omega(t))(1-2\nu)}. \tag{35}$$

Differentiating this equation in the case of a constant and exponential kernel, we obtain equations similar to equations (22) and (28) for the components of the vector damage parameter:

$$\frac{d\omega}{dt} = \frac{1-2\nu}{g-1+2\nu}(1-\omega), \tag{36}$$

$$\frac{d\omega}{dt} = \frac{1-2\nu-\alpha(g-1+2\nu)}{g-1+2\nu}(1-\omega). \tag{37}$$

Equations (36) and (37) are differential equations expressing the time-varying law of the damage vector projections for constant and exponential kernels, respectively. For the special case we are considering, we can write the absolute value of the damage vector using (29) as follows:

$$\varpi(t) = \sqrt{3}\,\omega(t). \tag{38}$$

As in the case of a simple stress state, for the failure process to occur in stages under a complex stress state, the condition $g-1+2\nu > 0$ must be satisfied. Otherwise, if $g-1+2\nu = 0$, $\frac{d\omega}{dt} = \infty$, which means that instantaneous failure occurs in the initial stages of loading. On the other hand, since the failure process in the material occurs under monotonous loading, the condition $\frac{d\omega}{dt} > 0$ must be satisfied. Therefore, for equation (36), the condition $(1-2\nu)(1-\omega) > 0$ must hold. From this, it follows that $\omega < 1$. Thus, there is a critical value $\omega = \omega_* < 1$ for which $\varpi(t) \approx 1$, meaning complete failure occurs. This corresponds to the brittle failure characteristic of the material. This property is similarly satisfied for equation (37).

## 3 Calculation of the quality of the damage process

Using the derived differential equations, the damage process was qualitatively analyzed. According to equations (36) and (37), numerical calculations were performed in the Matlab software package, and the damage accumulation process (in the initial condition $\omega(t = t_0) = 0$) was graphically depicted (Figure 1–4).

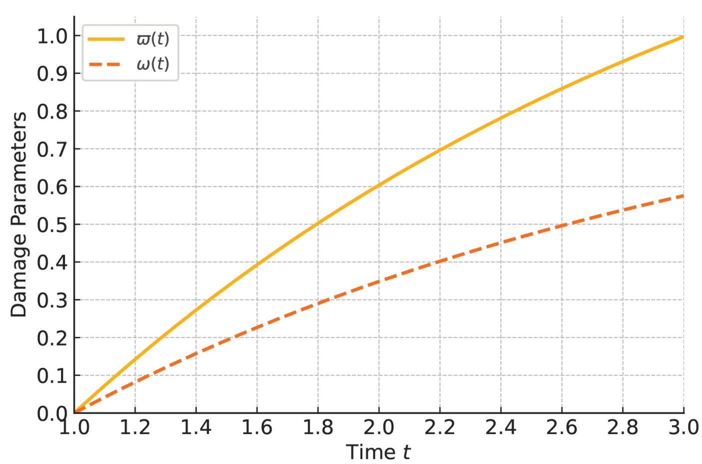


**Figure 1:** Damage accumulation model when $g = 2$ and $\nu = 0.2$

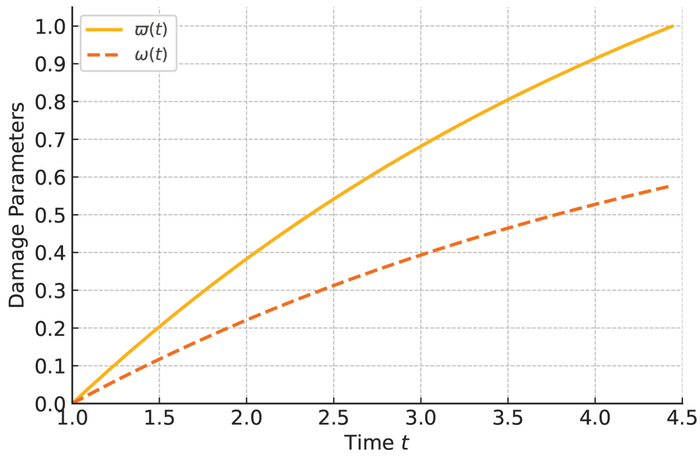


**Figure 2:** Damage accumulation model when $g = 2$ and $\nu = 0.3$

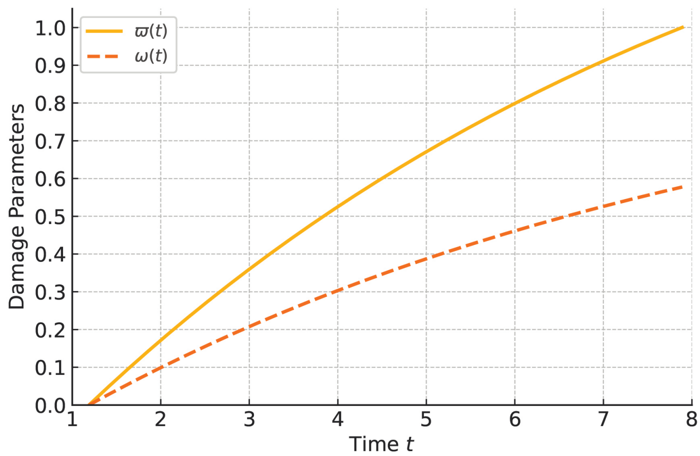


**Figure 3:** Damage accumulation model when $g = 2$, $\nu = 0.2$ and $\alpha = 0.3$

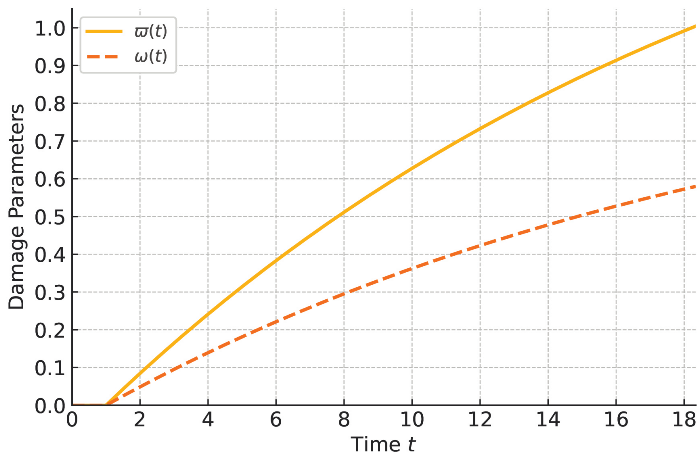


**Figure 4:** Damage accumulation model when $g = 2$, $\nu = 0.3$ and $\alpha = 0.2$

The analysis of the obtained results shows that for the constant kernel, the complete damage of the material occurs at $t^* = 3$ (Figures 1–2). First, considering the value $g = 2$ in equation

(19), the latent failure period (incubation period) of the material will be $t_0 = 1$. For $\nu = 0.2$, the critical values of the damage vector and its projections are approximately $\omega(t^*) \approx 0.5774$, $\varpi(t^*) \approx 1$. For $\nu = 0.3$, the complete damage of the material (critical time) will occur at $t^* = 4.44$. In this case, the critical values of the damage vector and its projections will be approximately $\omega(t^*) \approx 0.5768$ and $\varpi(t^*) \approx 1$.

For the exponential kernel, considering $g = 2$, $\nu = 0.2$, and $\alpha = 0.3$ in equation (20), the latent failure time (incubation period) of the material is $t_0 = 1.1889$. For these values, the solution of equation (37) gives the critical values of the damage vector and its projections as $\omega(t^*) \approx 0.5774$ and $\varpi(t^*) \approx 1$. In this case, the complete damage of the material occurs at $t^* = 7.89$.

Similarly, for $g = 2$, $\nu = 0.3$, and $\alpha = 0.2$, the complete damage of the material will occur at $t^* = 18.34$. At that time, the critical values of the damage vector and its projections will be approximately $\omega(t^*) \approx 0.5774$ and $\varpi(t^*) \approx 1$.

## 4 Conclusion

The problem of scattered damage of structural elements under simple and complex stress conditions was investigated using the strength criterion for hereditary-type damaged materials. The criterion-based approach was used to determine the initial damage time (incubation period), and differential equations were derived to describe the damage process for both simple and complex stress states. By comparing the constructed mathematical models with classical models, it was found that the time-dependent variation in the damage process is consistent with both models. However, the explicit inclusion of material constants in the newly derived differential equations can be considered an advantage.

The differential equations obtained for both stress conditions are linear, which allows for their analytical solution.

With the help of MATLAB software, the differential equations were solved for various material constant values, and curves characterizing the scattered damage process were constructed based on the obtained results.

These results can be used to predict the durability of various structural elements in nanosatellites operating under complex stress conditions.

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