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NEW TECHNIQUE FOR SOLVING NON-HOLONOMIC SYSTEMS

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Abstract. An optimization technique for approximate analytical equations is presented in this study, it is a different using for the Ezaki residual power series method (ERPS) to solve non- holonomic system problems that occur in a variety of physical phenomena. This method is based on the extended Taylor series formulation and the residue function. The positive results of the proposed methods have been shown that, it has been proven to be reliable, efficient and easy to use for solving all kinds of fractional non- holonomic system problems using Caputo derivative in technical and analytical fields. Moreover, our methodology has been shown positive and efficient results when compared with existing methods such as LRPS.

Keywords: Non-holonomic system, LRPS technique, Taylor series.
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1 Introduction

In recent decades, partially differential equations (PDEs) of fractional and integer ordering have become more and more popular as useful instruments for modeling. They are inspired by fluid dynamics difficulties, as well as other domains such as chemical control hypotheses, computational modeling, biotechnology, and multiple technical sectors (Debnath & Debnath, 2005).

Topics including singularity procedures, numerical techniques that operate with various PDE kinds, and functional analysis procedures are the main emphasis of the study (Abbasbandy, 2008). Some research uses fractional PDEs to investigate real-world issues in fluid mechanics and classical motion (Alomari et al., 2009).

One area of mathematical concepts that is thought to be a variant of classical computing is fractional calculus. Fractional calculus focuses on the principles that expansions and derivative for non-integer classes. In the past and in the present, the resulting derivative as well as the fractional integral were defined differently. Researchers have become interested in some of them because they have been utilized to demonstrate a variety of natural phenomena in a wide range of contexts, including the explanations supplied by (Ishteva, 2005) and (Jumarie, 2006).

Others, such as the idea of the adaptable (Abdeljawad et al., 2015) and the Caputo-Fabrizio modifications of fractions (Gambo et al., 2014), are still being investigated and explored in order to be incorporated into the theoretical structures of numerous occurrences in other scientific domains. Caputo's concept is the most widely accepted and authentic of the others since it

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emphasizes the starting value in actual situations and can be applied to a variety of situations. In international academic study, applications that utilize the fractional calculation concept so stand out as a notable example. Refer to (Kumar et al., 2023; Jawabreh et al., 2023) and (Singh & Bingi, 2024) for more details on FC. Numerous analytical and computing approaches are currently developed for solving nonlinear problems involving fractional orders (FDEs), while the majority of these problems lack specific analytical solutions (Guzman et al., 2021; Wang, 2013) and (Wang et al., 2019). The operational matrices procedures (Khan et al., 2020; Yu et al., 2019; Liaqat et al., 2023) and (Kumar, 2023), variational repetition (Li et al., 2023) and (Amir et al., 2024), Adomian disintegrate (Kareem, 2024), Partially Differential Progression (Yang et al., 2016) and (Zhao et al., 2023), Homotopy Assessment Approach (Khan et al.,2020), Homotopy perturbations approach (Yu et al., 2019) and (Liaqat et al., 2023), and Laplace transformation approaches (Kazem, 2013) and (Komashynska et al., 2016) are a few of these.

The Laplace method Fractional-order differential calculations can be handled using Sumudu, Elzaki, and numerous other transformations (Elzaki, 2012).

One of the modifications used to get a numerical representation of the commonly used partial differential equations which are used as computational representations of scientific occurrences is the Elzaki transformation (Liaqat et al., 2022).

In this area, we present, scientists have employed a range of transformations and tried-and-true techniques, such as non-integer demand log techniques (Ikram et al., 2019), non-integer ordering BBM-Burger formulations (Eriqat et al., 2023), and non-integer request relaxation-oscillation problems (Zhang et al., 2021). Numerous well-known mathematical issues requiring non-integer rank have been resolved using the Elzaki transformation mixing residue power series approach (ERPSM). The ERPSM was created to facilitate the analytical solution of non-integer order differential frameworks, which is essential for many applications in mathematics, science, and engineering. This particular effort's primary objective is to improve the accurateness of the dimensional procedure by solving it utilizing residual power series approaches and Elzaki transforms (El-Ajou, 2021) and (Pant et al., 2024).

Constraints including velocity measurements (or higher derivatives) will be added for for fractional non-holonomic structures using Caputo derivative. The following changes could be made to the calculation:

$$D^{\alpha}u(x,t) = u_{xx}(x,t) + \sigma u^3(x,t) + \delta(x,t,\dot{v}), \quad t > 0, \quad 0 < \alpha \leq 1, \tag{1}$$

and

$$u(x,t) = k(x,\dot{v}), \tag{2}$$

where $\delta(x,t,\dot{v})$ denotes the variables that rely on velocity and $\dot{v}$ indicates the velocity. The value of $u(x,t)$ is the standardized dispersion over a distance using the time delays t and D^{α} indicates the Caputo derivative of order $0 < \alpha \leq 1$.

2 Preliminaries

In this area, we present several concepts that are essential to our work.

Definition 1. *(Pant et al., 2024). The expression that follows can be utilized used to create the multifunctional value $\omega(x,\xi)$, where $\xi > 0$, which corresponds for the R-L time-fractional integrating given order $\alpha > 0$:*

$$J_{\eta}^{\alpha}\omega(x,\xi) = \frac{1}{\Gamma(\alpha)}\int_0^t (\xi-\tau)^{\alpha-1}\omega(x,\tau)\,d\tau, J_{\eta}^{0}\omega(x,\xi) = \omega(x,\xi). \tag{3}$$

Definition 2. *(Pant et al., 2024). The multifunctional value $\omega(x,\xi)$, where $\xi > 0$, corresponds to the Riemann-Liouville (R-L) time-fractional integral for the rank $\alpha \in (n-1,n]$, and is* *obtained by the following calculation:*

$$D_\eta^\alpha \omega(x,\xi) = \begin{cases} \frac{\partial^n}{\partial \xi^n} J_k^{n-\alpha} \omega(x,\xi), & n-1 < \alpha < n, \\ \frac{\partial^n}{\partial \xi^n} \omega(x,\xi), & \alpha = n. \end{cases} \tag{4}$$

Definition 3. *(Pant et al., 2024). The multivariate function $\omega(x,\xi)$, where $\xi > 0$, corresponding to the Caputo time-fractional integral for the rank $\alpha \in (n-1, n]$, is obtained by the subsequent calculation:*

$$D_\xi^\alpha \omega(x,\xi) = \begin{cases} J_\xi^{n-\alpha} \frac{\partial^n}{\partial \xi^n} \omega(x,\xi), & n-1 < \alpha < n, \\ \frac{\partial^n}{\partial \xi^n} \omega(x,\xi), & \alpha = n. \end{cases} \tag{5}$$

Definition 4. *(Pant et al., 2024). The Elzaki transform of a function $h(t)$ is represented by $E(\Omega(t))$, which can be expressed formally below:*

$$E(\Omega(t)) = u^2 \int_0^\infty \Omega(ut) e^{-t}\, dt, \quad u \in (\mathcal{Y}_1, \mathcal{Y}_2), \tag{6}$$

Lemma 1. *(Pant et al., 2024). The Fundamental Shapes of the Elzaki Transformation:*

1. $E(1) = \psi^2, \quad E(t) = \psi^3,$
2. $E(t^n) = n!\psi^{n+2},$
3. $E(e^{at}) = \frac{\psi^2}{1-a\psi}.$

Definition 5. *(Pant et al., 2024). The Elzaki transformation used to determine the fractional Derivative of Cap is defined as follows:*

$$E[D_t^\alpha \Omega(t)] = \mathbb{E}^{3-\alpha} E[\Omega(t)] - \sum_{\beta=0}^{n-1} \mathbb{E}^{2-\alpha+\beta} \beta^\beta(0), \tag{7}$$

where $1 - \frac{1}{n} < \alpha < 1.$

3 Innovative Approaches for the Non-Holonomic Systems in the ERPS Form

Using the fractional derivative expression $D^\alpha u(x,t)$ and the Elzaki transformation $D^\alpha u(x,t)$ in formulas (1) and (2), we obtain:

$$\Psi^{-\alpha} \mathbb{E}[u(x,t)] - \sum_{\xi=0}^{n-1} \Psi^{-\alpha+\xi} u^{(\xi)}(x,0) = \mathbb{E}\big[u_{xx}(x,t) + \sigma u^3(x,t) + \delta(x,t,\dot{v})\big]. \tag{8}$$

Elzaki transform applied to both positions:

$$\Psi^{-\alpha} \mathbb{E}[u(x,t)] - \sum_{\xi=0}^{n-1} \Psi^{-\alpha+\xi} u^{(\xi)}(x,0) = \Psi^2 \mathbb{E}[u_{xx}(x,t)] + \sigma \mathbb{E}[u^3(x,t)] + \delta(x,t,\dot{v}). \tag{9}$$

Given that:

$$\mathbb{E}[u_{xx}(x,t)] = \mathbb{E}\left[\frac{\partial^2 u(x,t)}{\partial x^2}\right],$$

the Elzaki transform for this extra derivative component would adhere to the same transformational principles as $\frac{\partial^2 u(x,t)}{\partial x^2}$. As a result, the modified equation is:

$$\Psi^{-\alpha}\mathbb{E}[u(x,t)] - \sum_{\xi=0}^{n-1}\Psi^{-\alpha+\xi}u^{(\xi)}(x,0) = \Psi^2\mathbb{E}[u_{xx}(x,t)] + \sigma\mathbb{E}[u^3(x,t)] + \delta(x,t,\dot{v}) \tag{10}$$

Use the Elzaki transform's reverse to go back to the period of the time field:

$$\mu(x,t) = \sigma^{-1}\left[\Psi^{-\alpha}\mathbb{E}[u(x,t)] - \sum_{\xi=0}^{n-1}\Psi^{-\alpha+\xi}u^{(\xi)}(x,0)\right] \tag{11}$$

Consider the next series expression for $u(x,t)$:

$$u(x,t) = \sum_{n=0}^{\infty}\Omega_n(x)t^{(n+1)} \tag{12}$$

To calculate the residual, enter the following sequence within the original formula. At every phase, the series of expansion's residue phrase is provided by:

$$Res(x,t) = u(x,t) - \Omega(x) - e^{-1}\left[\Psi^{-\alpha}\mathbb{E}[u_{xx}(x,t)] + \sigma\mathbb{E}[u^3(x,t)] + \delta(x,t,\dot{v})\right]. \tag{13}$$

For every term in a series, the residuals are calculated one after the other. Beginning with $n=0$, use the residual at $n=0$ to determine $\Omega_0(x)$:

$$Res_0(x,t) = D^{\alpha}\Omega_0(x) - \Omega_0''(x) - \sigma\Omega_0^3(x) - \delta(x,t,\dot{v}). \tag{14}$$

Next, compute $\Omega_1(x)$ in the following way:

$$\Omega_1(x) = \frac{1}{1!}Res_0(x,t)\big|_{t=0}. \tag{15}$$

Iterate further for larger values of n:

$$Res_1(x,t) = D^{\alpha}(\Omega_0(x)+\Omega_1(x)t^n) - (\Omega_0(x)+\Omega_1(x)t^n)'' - \sigma(\Omega_0(x)+\Omega_1(x)t^n)^3 - \delta(x,t,\dot{v}). \tag{16}$$

Following simplification at $t=0$, find $\Omega_2(x)$ using:

$$\Omega_2(x) = \frac{1}{2!}Res_1(x,t)\big|_{t=0}. \tag{17}$$

Similarly, for $n=3,4,\ldots$, we can obtain of $\Omega_3(x)$ and $\Omega_4(x)$ as same way. The total terms for the series expanding yields the final approximations of the outcome:

$$u(x,t) \approx \sum_{n=0}^{N}\Omega_n(x)t^n, \tag{18}$$

where N is the quantity of terms required to reach the required level of precision.

4 Application

Let's say that the preliminary formula was:

$$D^{\alpha}u(x,t) = u_{xx}(x,t) + 3u^4(x,t) + \delta(x,t,\dot{v}), \quad t>0,\ 0<\alpha\le 1, \tag{19}$$

and the first requirement is:

$$u(x,0) = \sin x, \tag{20}$$

where the velocity-dependent constraints are represented by $\delta(x,t,\dot{v})$.

The following is the application of the Elzaki transform to the fractional derivatives:

$$E[D^{\alpha}u(x,t)] = \Psi^{-\alpha}\mathbb{E}[u(x,t)] - \Psi^{-\alpha}u(x,0) \tag{21}$$

Consequently, putting this into the calculation of (19), we get:

$$\Psi^{-\alpha}\mathbb{E}[u(x,t)] - \Psi^{-\alpha}\sin x = \mathbb{E}[u_{xx}(x,t)] + 3\mathbb{E}[u^4(x,t)] + \mathbb{E}[\delta(x,t,\dot{v})], \tag{22}$$

whenever $\mathbb{E}[u(x,t)]$, then,

$$\mathbb{E}[u_{xx}(x,t)] = \Psi^2\mathbb{E}[u(x,t)].$$

For $\mathbb{E}[\delta(x,t,\dot{v})]$, it becomes more difficult to apply the Elzaki transformation to this term because $\delta(x,t,\dot{v})$ is velocity-dependent.

$$\Psi^{-\alpha}\mathbb{E}[u(x,t)] - \Psi^{-\alpha}\sin x = \Psi^2\mathbb{E}[u(x,t)] + 3\mathbb{E}[u^4(x,t)] + \mathbb{E}[\delta(x,t,\dot{v})]. \tag{23}$$

Modify the formula so that $\mathbb{E}[u(x,t)]$ is isolated on one side by $\mathbb{E}[u(x,t)]$:

$$\Psi^{-\alpha}\mathbb{E}[u(x,t)] - \Psi^{-\alpha}\sin x = \Psi^2\mathbb{E}[u(x,t)] - 3\mathbb{E}[u^4(x,t)] - \mathbb{E}[\delta(x,t,\dot{v})]. \tag{24}$$

Suppose that a series expanding in powers of t may be used to represent the outcome:

$$u(x,t) = \sum_{j=0}^{\infty}\Omega_j(x)t^j. \tag{25}$$

To begin, compute the residual for $n=0$:

$$Res_0(x,t) = D^{\alpha}\cos x - (\sin x)_{xx} - 3\sin^4 x - \delta(x,t,\dot{v}). \tag{26}$$

Given that $\sin'' x = -\sin x$, we obtain:

$$Res_0(x,t) = D^{\alpha}\sin x + \sin x - 3\sin^4 x - \delta(x,t,\dot{v}). \tag{27}$$

The residue thus changes to:

$$Res_0(x,t) = D^{\alpha}\sin x - 3\sin^4 x - \delta(x,t,\dot{v}). \tag{28}$$

Applying the given equation for $\Omega_1(x)$, we obtain:

$$\Omega_1(x) = \frac{1}{\Gamma(1+\alpha)}Res_0(x,t)\big|_{t=0}. \tag{29}$$

Replace the formula for $Res_0(x,t)$ at $t=0$:

$$\Omega_2(x) = \frac{1}{\Gamma(1+\alpha)}\left(\sin x - 3\sin^4 x - \delta(x,t,\dot{v})\right). \tag{30}$$

Following that, find the residual at $n=1$:

$$Res_1(x,t) = D^{\alpha}(\Omega_0(x)+\Omega_1(x)t^n) - (\Omega_0(x)+\Omega_1(x)t^n)_{xx} - 3(\Omega_0(x)+\Omega_1(x)t^n)^4 - \delta(x,t,\dot{v}). \tag{31}$$

For $t=0$, compute $\Omega_2(x)$:

$$\Omega_2(x) = \frac{1}{\Gamma(1+2\alpha)}Res_1(x,t)\big|_{t=0}. \tag{32}$$

Subsequently, apply the same formula to determine $\Omega_2(x)$:

$$\Omega_2(x) = \frac{1}{\Gamma(1+2\alpha)} Res_1(x,t)\big|_{t=0}. \tag{33}$$

Likewise, determine the residual for $\Omega_3(x)$:

$$\begin{aligned} Res_2(x,t) =& D^\alpha\big(\Omega_0(x) + \Omega_1(x)t^\alpha + \Omega_2(x)t^{2\alpha}\big) \\ &- \big(\Omega_0(x) + \Omega_1(x)t^\alpha + \Omega_2(x)t^{2\alpha}\big)_{xx} \\ &- 3\big(\Omega_0(x) + \Omega_1(x)t^\alpha + \Omega_2(x)t^{2\alpha}\big)^4 - \delta(x,t,\dot{v}) \end{aligned} \tag{34}$$

When $t = 0$, this reduces to:

$$Res_2(x,t)\big|_{t=0} = D^\alpha\Omega_2(x) - \Omega_2(x)'' - 12\sin^3 x\Omega_2(x) - \delta(x,t,\dot{v}). \tag{35}$$

Next, we calculate $\Omega_3(x)$ as follows:

$$\Omega_3(x) = \frac{1}{\Gamma(1+3\alpha)} Res_2(x,t)\big|_{t=0}. \tag{36}$$

Every parameter in the series expansion adds up to the final answer for $u(x,t)$:

$$u(x,t) = \sin x + \sum_{j=0}^{\infty} \Omega_j(x) \frac{t^{j\alpha}}{\Gamma(1+j\alpha)}. \tag{37}$$

The following would be the initial terms of this expanding:

$$\begin{aligned} u(x,t) = \sin x &- \frac{\sin x - 3\sin^4 x - \delta(x,0,\dot{v})}{\Gamma(1+\alpha)} t^\alpha \\ &- \frac{\sin 2x - 5\sin 6x - \delta(x,0,\dot{v})}{\Gamma(1+2\alpha)} t^{2\alpha} \\ &- \frac{\sin 3x - 7\sin 8x - \delta(x,0,\dot{v})}{\Gamma(1+3\alpha)} t^{3\alpha} + ... \end{aligned} \tag{38}$$

Considering different values of α in an area of $(0,\infty) \times [0, 0.01]$, Tables 1 and 2 show the numerical approaches to the issue, the third estimates outcome for various values of α, and the accompanying residual and relative errors. The outcomes show that a fractional calculation can be computationally solved with a high degree of reliability using the ERPS approach.

Graphical representations of the third estimated outcomes for constructions (19) with (20) over the interval $(0,\infty) \times [0, 0.01]$ are shown in Figures 1a, b, c, and d. The statistics show that the responses for IVPs (19) and (20) are continuously decreasing across the area.

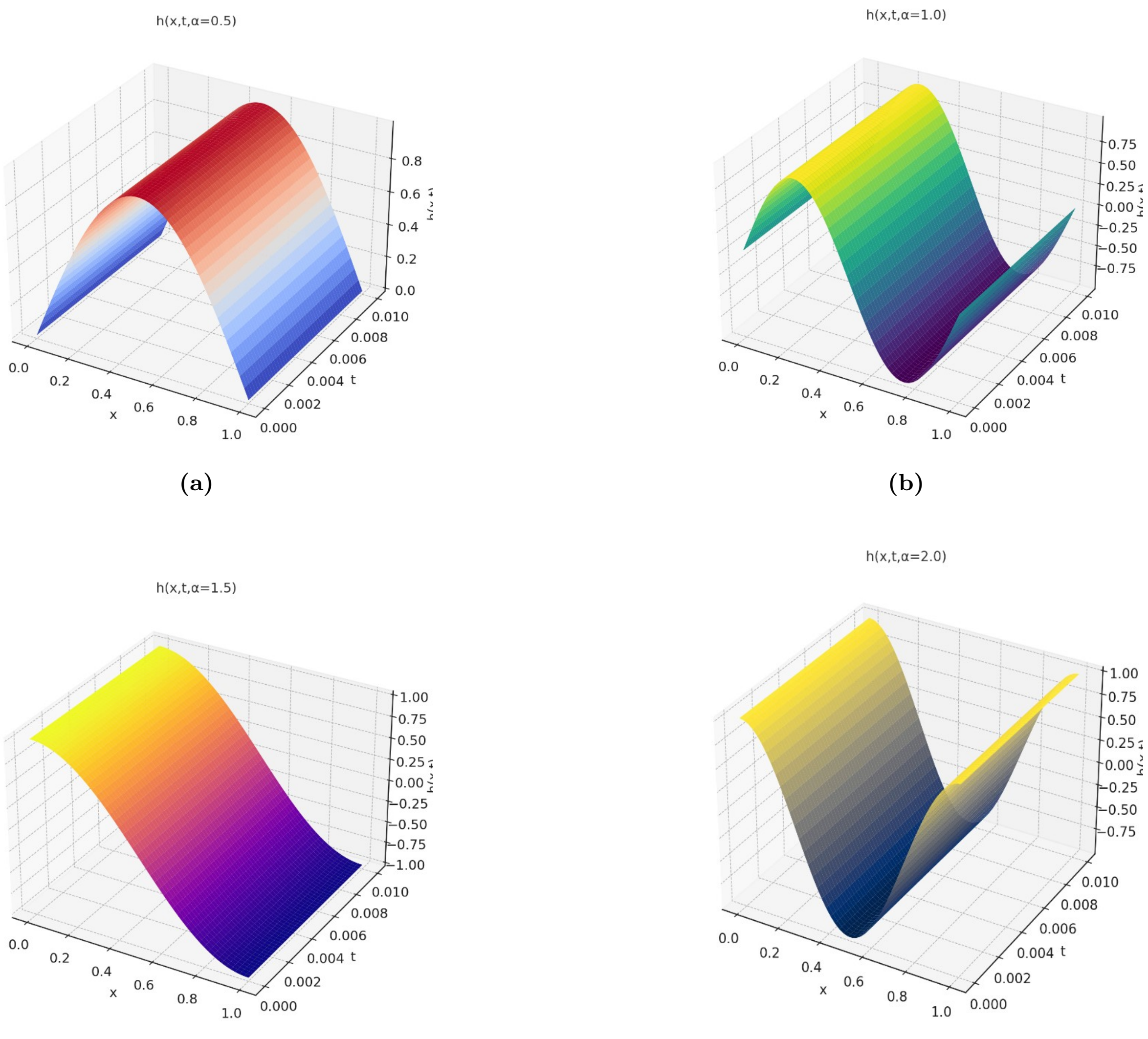
Table 1: Evaluating the ERPS, LRPS, and residual remedies at $\alpha = 0.5$ with values $x = [0, 1], t = 0.01$.

X	ERPS	LRPS	Residual results
0.0	-0.00166	0.000193	0.001106
0.1	0.004906	0.001951	0.003725
0.2	-0.008999	-0.005218	0.005287
0.30000000000000004	-0.002459	-0.007185	0.006035
0.4	-0.005078	-0.003374	4.5e-05
0.5	-0.005679	-0.006506	-0.001017
0.6000000000000001	-0.00332	-0.002733	-0.000734
0.7000000000000001	0.00034	-0.003256	0.005109
0.8	0.00183	-0.001189	0.001412
0.9	0.005126	0.008133	0.002692
1.0	0.003178	0.007861	0.006514

Table 2: Evaluating the ERPS, LRPS, and residual remedies at $\alpha = 0.2$ with values $x = [0, 1], t = 0.01$.

x	ERPS	LRPS	Residual results
0.0	-0.00128	-0.000989	4e-06
0.1	-0.007495	-0.011149	0.00204
0.2	0.004887	0.005023	-0.001356
0.30000000000000004	0.004353	0.001197	0.003543
0.4	0.005581	0.008434	0.001757
0.5	0.005021	0.008561	0.001968
0.6000000000000001	0.003413	0.003356	-0.001061
0.7000000000000001	0.01224	0.015706	0.002865
0.8	0.005989	0.001785	0.004075
0.9	0.005075	0.005127	-0.001141
1.0	0.011516	0.007169	0.004909

Furthermore, the predicted result's numerical representations closely resemble the actual method, as the adjacent table demonstrates.



(a) (b) (c) (d)

Figure 1: The plots of formulas illustrate a wide range of outcomes for α: (a) $\alpha = 0.5$, (b) $\alpha = 1$, (c) $\alpha = 1.5$, and (d) $\alpha = 2$

5 Conclusion

This paper presents a new and reliable method known as ERPS for solving numerically defined fractional problems. This strategy is an improvement over the traditional method known as the RPSM, combining the ET and the RPS. This technique is less complex in terms of calculations and provides solutions with fewer errors. A sequential step was followed to determine the values of the coefficients using the results of this series, and the method proved to be efficient in solving specialized fractional equations with exceptional accuracy, thanks to a consistent computational methodology. Compared to the traditional LRPS method through illustrative tables, the ERPSM method proved to be able to provide accurate results that are equal to, and in some cases even superior to, the traditional method. In addition, this method provides an accurate and simple solution for estimating solutions to specialized quadratic and non-holonomic fractional problems. Thus, this methodology is an effective and practical tool for solving complex fractional systems accurately and quickly.

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