Advanced Physical Research
Vol.7, No.3, 2025, pp.315-321
https://doi.org/10.62476/apr.73315

STUDY OF 0^+ STATES IN NUCLEI INCLUDING SPIN-ORBIT FORCES

S.G. Abdulvahabova[1], **I.G. Afandiyeva**[2*], **D.J. Askerov**[2], **A.L. Baxtiyarov**[2]

[1]Baku State University, Baku, Azerbaijan
[2]Azerbaijan State University of Oil and Industry, Baku, Azerbaijan

Abstract. The excitation of 0^+ states in direct (p,t) nuclear reactions is investigated with consideration of the spin-orbit interaction. The energies of 0^+ states and the cross sections of (p,t) reactions for selected isotopes of samarium (Sm) and gadolinium (Gd) have been studied. The spin-orbit force influences the energies of excited 0^+ states by altering the nuclear density distribution, contributing to configuration mixing, and affecting the overall nuclear shape. Theoretical calculations demonstrate that the energies of the lowest 0^+ than the corresponding experimental values. Furthermore, the spin-orbit interaction leads to an increase in the effective cross sections observed in (p,t) reactions.

Keywords: direct nuclear reaction, spin-orbital interaction, cross section, excited 0^+ states.
[*]**Corresponding author:** I.G. Afandiyeva, Azerbaijan State University of Oil and Industry, Baku, Azerbaijan, e-mail: *afandiyeva.irada@asoiu.edu.az*
Received: 5 July 2025; Revised: 26 August 2025; Accepted: 12 September 2025; Published: 16 October 2025.

1 Introduction

In deformed even-even nuclei of the rare-earth region, a large number of energy levels with spin and parity 0^+ are observed in direct (p,t) and (t,p) reactions. The study of 0^+ states populated in (p,t) and (t,p) reactions is of considerable interest. These reactions are important tools for analyzing the structure of collective states and nuclear dynamics, as they are highly sensitive to the collective properties of nuclei (Boltushkin & Nikiforov, 2007). Nevertheless, experiments involving (p,t) and (t,p) reactions are conducted less frequently, since tritium is a radioactive isotope of hydrogen and requires special storage conditions. It has a low momentum, which complicates its detection. (p,t)-type reactions also have relatively small cross sections. To obtain acceptable statistics, it is necessary to use intense proton beams and conduct measurements over extended periods.

In rare-earth nuclei, two to three 0^+ states have been found experimentally below the energy gap (Meyer et al., 2006; Kulp et al., 2003; 2008; Heyde & Wood, 2011; Wood, 2012). The description of the energies of the excited 0^+ states provides an important test for evaluating the applicability of various nuclear structure models. By analyzing these states, certain conclusions can be drawn about the types of excited 0^+ states. The energy of these states is lower than the energy gap, which indicates that they are not pairing vibrations. Furthermore, the regular population of 0^+ states in direct (p,t) and (t,p) reactions in rare-earth and actinide nuclei suggests that these states possess a pronounced collective nature. On the one hand, 0^+ states can be multi-phonon, quasi-particle–phonon, or mixed residual states. In work discussed the competition between the collectivity and the pairing correlation in the states populated by the two transferred particles. The theoretical results show that the states with high collectivity are preferably accessed by the sequential two-neutron transfer process suggesting that the correlation of the two transferred neutrons was broken by the collectivity of the populated final state. In the work (Lubian et al., 2019), the competition between collectivity and pairing correlations in states populated by two-neutron transfer was discussed. The theoretical results show that states with strong collectivity are preferentially populated via a sequential two-neutron transfer mechanism, suggesting that the correlation between the transferred neutrons is disrupted by the collective nature of the final state. The work Quan et al. (2017) presents a detailed theoretical investigation of the nature of the excited 0^+ states in ^{158}Gd using relativistic nuclear models.

The properties of excited 0^+ states generated by pairing and quadrupole -quadrupole forces in samarium isotopes were investigated in the work (Abdulvahabova & Afandiyeva, 2025). With an appropriate choice of the quadrupole -quadrupole interaction parameter, the second excited state appears just below the energy gap.In (Afandiyeva & Ahmedov, 2025), the properties of excited 0^+ states were studied with consideration of Coulomb interactions in rare-earth nuclei. The Coulomb potential significantly influences the effective cross sections of (p,t) reactions, particularly at low energies, where it suppresses proton–nucleus interactions.

In the work Abdulvahabova & Afandiyeva, (2021), it was shown that in two-nucleon transfer reactions, 0^+ levels are more strongly excited in deformed nuclei where the single-particle quadrupole moments near the Fermi surface have the same sign. This leads to the conclusion that residual interactions - the part of the interaction that cannot be reduced to the mean field, play a significant role.

It should be noted that in all theoretical studies, the calculated values for the characteristics of the 0^+ states (excitation energy, transition probabilities $E2$ and $\rho(0)$, Rasmussen parameter) and the effective cross-sections of the (p,t) and (t,p) reactions do not quantitatively agree with the experimental data.

In this paper, we investigate the 0^+ states populated in (p,t) reactions within the framework of the cluster model, taking into account the residual spin-orbit interaction. The objective of this study is to investigate the influence of the residual spin-orbit interaction on the 0^+ states populated in (p,t) reactions. The analysis is carried out within the framework of a cluster model, and the results are applied to selected isotopes of samarium and gadolinium.

2 Expression for the Effective Cross Section

Atomic nuclei can be regarded as systems composed of clusters of various types. In the cluster model, the structure of the nucleus is represented as various configurations of nucleons and clusters consisting of several particles (Brejnoy & Mikhalyuk, 2008; Erban et al., 2014; Freer, 2010; Neudachin et al., 2005).

The cluster model allows the expression of the many-nucleon nucleus wave function to be reduced to the expression of the single-particle wave function of its constituent clusters. The binding energy of a cluster is relatively small, and this fact enables the development of a theory for treating continuum states as accurately as bound states.

The interaction of clusters with the other nucleons in the nucleus is described by realistic interactions, using various methods. Realistic interactions are typically characterized by two-body potentials, and they not only describe the properties of the ground state but also predict the emergence of much more pronounced cluster structures. Based on realistic nucleon-cluster interactions, it is possible to account for the contribution of low-lying excited states of nuclei, since at medium and intermediate energies of the incoming particle, excitation of the system's low-lying states usually occurs.

The cluster model is widely used to explain the mechanism of direct nuclear reactions and cluster decay process (Berezhnoy & Mikhailyuk, 2008; Kadmenskya et al., 2007; Neudachin et al., 2005). In this model, the internal structure of each cluster is neglected, and the cluster is treated as an elementary particle.

According to the cluster model, in (p,t) reactions, the transferred neutrons exist in the nucleus as clusters. This means that the reaction mechanism involves the direct interaction of the incoming proton with a neutron cluster a dineutron. According to this model, the neutrons transferred in (p, t) reactions exist in the nucleus as associations. The collision of a proton with the nucleus, whose de Broglie wavelength is significantly smaller than the average inter nucleon distance in the nucleus, leads to the emergence of clusters in the final state, whose kinetic energy exceeds the binding energy of nucleons in these clusters by tens or even hundreds of times. This means that the reaction mechanism involves the direct interaction of the incoming proton with nucleon associations. The dineutron association differs from free nucleons because it is surrounded by other nucleons. This surrounding field alters the properties of the association compared to free neutrons.

Assume that a dineutron cluster, denoted as X, is already embedded within the nucleus A i.e., $A = A_1 + X$, and a proton collides with the nucleus. The proton captures the cluster X and is emitted as a triton-t, forming nucleus B. At proton energies of several tens of MeV, the interaction between the proton and the cluster X occurs at the periphery of the nucleus. In addition, it is assumed that one-step cluster capture dominates over uncorrelated two-neutron capture.

When constructing the wave functions, it is essential to choose the correct asymptotic behavior, since transfer reactions predominantly occur in the peripheral region of the nucleus. In all expressions, the momenta and coordinates of the corresponding particles and nuclei are denoted by k and ξ, respectively.

The wave function of the initial state is given by:

$$\Psi_i = e^{ik_p r}\Psi_A\left(\xi_{A_1}, \xi_X, \rho\right), \tag{1}$$

where $e^{ik_p r}$ - wave function of incident proton and

$$\Psi_A\left(\xi_{A_1}, \xi_X, \rho\right) = \Psi_{A_1}\left(\xi_{A_1}\right)\Psi_X\left(\xi_{\mathrm{X}}\right)\Psi_{A_1 X}\left(\rho\right), \tag{2}$$

where $\Psi_{A_1}(\xi_A)$ and $\Psi_X(\xi_X)$ are the internal wave functions of the residual nucleus and the dineutron cluster, respectively, and $\Psi_{A_1 X}(\rho)$ describes their relative motion. The relative coordinate ρ is given by the formula (Abdulvahabova & Afandiyeva, 2025)

$$\rho = \frac{1}{A_1}\sum_{i=1}^{A_1} r_i - \frac{1}{X}\sum_{j=1}^{X} r_j. \tag{3}$$

The final state wave function has the following form

$$\Psi_f = e^{ik_t r_t}\Psi_B(\xi_B). \tag{4}$$

Here, $e^{ik_t r}$ denotes the wave function of the free triton, and $\Psi_B(\xi_B)$ represents the internal wave function of the final nucleus B.

We limited ourselves to the assumption that the internal states of the incoming triton, the outgoing proton, and any intermediate associations are fully symmetric S-states. This implies that the corresponding successive transfer interactions are diagonal in the spin states of the nuclei. Under this assumption, the radial wave functions of the two neutrons forming the association captured by the nucleus are close to each other, particularly in the peripheral region of the nucleus where the wave functions of these nucleons in the target nucleus A and in the resulting nucleus B "match." If the energies E_1 and E_2 of the first and second stripped neutrons are close to each other, then these radial functions strongly overlap and nearly replicate each other, resulting in the capture of a "correlated" pair of pair of nucleons.

We will choose the potential V beetwen the proton and nucleus A in the following form:

$$V = V_N + V_{ls}. \tag{5}$$

In (5) V_N represents the central nuclear potential, described by the Woods-Saxon form

$$V_N = \frac{-V_0}{1+\exp\left(r-R_0\right)/\alpha}, \tag{6}$$

and V_{ls} denotes the residual spin-orbit interaction potential.

The residual spin-orbit interaction represents a deviation from the mean field and constitutes a part of the two-body interaction that manifests itself in excited states and in pairing correlations. The spin-orbit potential influences both the energy and the collectivity of excited 0^+ states by altering the potential energy surface of the nucleus, leading to the stabilization of deformed configurations. Additionally, the spin-orbit interaction modifies the energy spectrum single-particle levels, which in turn affects the mixing of configurations, particularly the interplay between non-collective and collective states. (p,t) reactions are also sensitive to configurations characterized by different total angular momenta j, which makes them a useful tool for probing the effects of the spin-orbit interaction.

Two-particle spin-orbit force are usually written in the form

$$G_{ls} = -\frac{k_{sl}}{2}\left(\vec{\sigma}_1+\vec{\sigma}_2\right)\left(\vec{p}_1-\vec{p}_2\right)\Delta_1\delta(\vec{r}_1-\vec{r}_2), \tag{7}$$

where k_{sl} is the spin-orbit interaction constant, $\vec{\sigma}$- Pauli matrix and $\vec{p}$ are the momenta of the interacting particles. The force parameter k_{sl} estimated based on the splitting energy of spin-orbit doublets in the nuclear potential. The contribution (7) to the self-consistent nuclear field leads to the splitting of single-particle levels according to angular momentum $j = l \pm 1/2$:

Within the framework of the Born approximation, the matrix element and the cross section of the (p,t) reaction take the following form:

$$M_{i\to f} = \int e^{ik_t r_t}\Psi_B^*(\xi_B)Ve^{-ik_p r_p}\Psi_{A_1}(\xi_{A_1})\Psi_X(\xi_X)\Psi_{A_1X}(\rho)d\xi_{A_1}d\xi_X d\xi_B d\rho dr_p dr_t. \tag{8}$$

The effective cross section for the (p,t) reaction in this case can be written as

$$d\sigma = \frac{E_pE_A}{\lambda^{1/2}(m_p^2+m_A^2)(2J_p+1)}\int(2\pi)^4\delta\left(p_p+p_A-p_t-p_B\right)\left|M_{i\to f}\right|^2 dp_t, \tag{9}$$

here, $E_p = \left(p_p^2+m_p^2\right)^{1/2}$ denotes the energy of the incoming proton, $E_A = \left(p_A^2+m_A^2\right)^{1/2}$ is the energy of the target nucleus, and $\lambda(x,y,z) = (x-y-z)^2-4yz$ is the kinematic function arising from energy and momentum conservation.

3 Analysis of Results

The calculation results and experimental data for several samarium and gadolinium isotopes are summarized in the Table 1. In the work Afandiyeva & Ahmedov (2025), the properties of excited 0^+ states were investigated with the inclusion of the Coulomb interaction. For comparison, Table 1 presents the results of calculations that account for the Coulomb interaction (Theory 1) alongside the results obtained using the approach developed in the present work (Theory 2).

Experimental data are taken from the works Kulp et al. (2003; 2008); Meyer et al. (2006).

The spin-orbit interaction constant can be estimated from the energy of the spin-orbit splitting of single-particle levels, i.e., from the condition:

$$< H_{sl} >=< H_{sl}^{s.p.} >, \tag{10}$$

where

$$H_{sl}^{s.p} = -k_{sl} < i\left|(\sigma l)\frac{dV(r)}{rdr}\right|j >, \tag{11}$$

Table 1: Calculation results for the energy of 0^+ states and the cross sections of (p,t) reactions

	Nucleus	Sm^{152}			Sm^{154}			Gd^{154}			Gd^{156}		
Theory 1	ω(MeV)	0,68	2,55	3,12	1,10	2,45	2.89	0,68	2,15	2,34	1,05	2,15	2,30
	$\sigma(p,t)/\sigma_0$	1,26	0,13	0,03	1,23	0,05	0,02	1,23	0,12	0,01	1,09	0,03	0,01
Theory 2	ω(MeV)	0,68	1,59	1,87	1,10	1,65	2,05	0,68	1,56	2,06	1,05	1,67	1,89
	$\sigma(p,t)/\sigma_0$	1,.68	0,02	0,01	1,23	0,04	0,01	2,35	0,01	0,01	0,43	0,14	0,01
Experiment	ω(MeV)	0,68	1,09	1,66	1,10	1,22	-	0,68	1,29		1,05	1,17	1,71
	$\sigma(p,t)/\sigma_0$	0,28	0,01	-	-	-	-	0,13	-		0,11	-	-

is the spin-orbit term in the self-consistent mean-field potential. $V(r)$ is taken as a function of the nuclear density distribution, typically of the Woods–Saxon type. A variation in $V(r)/dr$, which is nonzero only at the nuclear surface, leads to a redistribution of nucleons among different orbital angular momenta l.

For comparison with experimental data, calculations were performed using the following parameter values:

$$k_{sl} = k_0 \cdot 10^4 A^{-4/3} fm^5, \quad k_0 = 0,28 \quad for\, A \approx 155, \tag{12}$$

where the constant k_o was chosen so that the energy of the first excited state coincided with the experimental value. We assume that the values k_{sl} are the same for the neutron and proton systems. In self-consistent fields with the Woods-Saxon potential, the spin-orbit interaction constant without including isospin asymmetry is usually chosen to be the same for both protons and neutrons.

The parameters of the Woods–Saxon potential were adopted from Schwierz et al. (2007).

The Coulomb potential influences both the energy and the collectivity of excited 0^+ states by altering the potential energy surface of the nucleus, leading to the stabilization of deformed configurations. Additionally, the Coulomb interaction modifies the energy spectrum of proton single-particle levels, which in turn affects the mixing of configurations particularly the interplay between normal (non-collective) and collective states. The inclusion of the long-range Coulomb interaction in addition to the short-range nuclear potential leads to substantial changes in the asymptotic behavior of the wave functions, which in turn affects the analytical properties of the scattering amplitude.

The spin-orbit interaction influences the energies of excited 0^+ states through modifications in the nuclear density distribution, configuration mixing, and nuclear shape. The comparison of Theory 1 and Theory 2 shows that spin-orbit potential has a significant effect on the energy of 0^+ states and effective cross section of (p,t) reactions, particularly at low energies. However, as seen from the table, the second excited state does not drop below the two-particle excitations (below the energy gap 2Δ).

4 Conclusion

The inclusion of residual spin-orbit interactions, like other theoretical approaches, does not yield quantitatively accurate results. Based on the present study, it can be concluded that the theoretical predictions for these states differ significantly from the available experimental data. This discrepancy can be attributed to the simplifying assumptions inherent in the model. Addressing these discrepancies requires not only a refinement of the mean-field approximation but also the exploration of new mechanisms responsible for the generation of collective excitations in nuclei.

One contributing factor is the fact that the interaction between the proton and the dineutron cluster does not occur in free space but within the field of a heavy nucleus, which absorbs part of the transferred momentum. This effect complicates the reaction dynamics. In the present work, it was assumed that no momentum is transferred to nucleus B.

Additionally, the contribution of incoherent rescattering processes specifically, the interaction of the proton with other dineutron clusters within the target nucleus A was not accounted for. Furthermore, in two-nucleon transfer reactions, unlike in simpler single-nucleon transfer processes, it is not possible to extract the spectroscopic amplitude directly from the experimental cross section due to interference effects.

Accurate cross-section estimation thus requires the ability to calculate the overlap integral within a reliable nuclear structure model. For low-lying collective states, spectroscopic amplitudes can be reliably predicted only in limiting cases when the nuclei involved are either spherical or strongly deformed. In contrast, no satisfactory theoretical formalism currently exists for nuclei in the transitional region.

Thus, none of the interactions proposed for the description of excited 0^+ states can be reasonably regarded as exclusively determining or dominant. The population of these states in rare-earth nuclei indicates their pronounced collective nature. This observation highlights the necessity of further investigation into new mechanisms responsible for the generation of collective nuclear excitations. In this context, particular interest lies in the study of interactions characterized by fundamentally different symmetry properties.

At present, theoretical approaches based on the energy density functional have attracted significant attention (Joulos & Kolganova, 2024). Within this framework, the problem of self-consistent description of both the nuclear mean field and residual interactions is addressed. Both non-relativistic functionals and relativistic approaches based on a Lagrangian involving nucleon–meson interactions are employed.

Therefore, empirical data available for theoretical development are quite limited. Unfortunately, experiments involving (p,t) and (t,p) reactions are conducted infrequently, as tritium being a radioactive isotope of hydrogen requires special handling and storage conditions. Its low momentum further complicates detection. Moreover, (p,t) reactions are characterized by relatively small cross sections. Consequently, achieving statistically significant results necessitates the use of high-intensity proton beams and prolonged measurement durations.

References

Abdulvahabova, S.G., Afandiyeva, I.G. (2025). Study of 0^+ excitations in nuclei taking into account residual forces. *Izv. Vuz. Fizika*, 68, 74-80, `https://doi.org/10.17223/00213411/68/3/10`.

Abdulvahabova, S.G., Afandiyeva, I.G. (2021). Population of 0^+ excited states in two-nucleon transfer reactions. *Izv. Vuz. Fizika 64*(3), 121-126. `https://doi.org/10.17223/00213411/64/3/121`

Afandiyeva, I.G., Ahmedov, R.A. (2025). Study of 0^+ states in nuclei with account of spin-quadrupol forces. *Journal of Physics & Space Sciences*, 2(1), 85-89, `https://doi.org/10.30546/209501.101.2.300.021`.

Berezhnoy, Y.A., Mikhailyuk, V.P. (2008). Alpha clustering in the scattering of particles and light atomic nuclei on nuclei. *Physics of Particles and Nuclei, 39*(2), 221-258.

Boltushkin, A.Ya., Nikiforov, N.N. (2007). Nuclear reactions (p, t) and (t, p) in the study of collective modes in nuclei. *Physics of Atomic Nuclei*, 70(9), 1501-1510.

Heyde, K., Wood, J.L. (2011). Shape coexistence in atomic nuclei. *Review of Modern Physics*, 83, 1467-1521.

Freer, M. (2010). Alpha-cluster model of atomic nuclei. *Scholarpedia, 5*(6), 9652.

Lubian, J., Ferreira, J.L., Linares, R., Cardozo, E.N., Paes, B., Cappuzzello, F., ... & Lichtenthäler, R. (2019). Role of correlations in two-neutron transfer reactions. In *EPJ Web of Conferences* (Vol. 223, p. 01035). EDP Sciences. https://doi.org/10.1051/epjconf/201922301035

Jolos, R.V., Kolganova, E.A. (2024). Structure of the Atomic Nucleus: Yesterday and Today. *Physics of Particles and Nuclei, 55*(5), 1209-1221.

Kadmensky, S. G., Kurgalin, S. D., & Tchuvil'sky, Y. M. (2007). Cluster states in atomic nuclei and cluster-decay processes. *Physics of Particles and Nuclei, 38*(6), 699-742. https://doi.org/10.1134/S1063779607060019

Kulp, W.D., Wood, J.L., Krane, K.S., Loats, J., Schmelzenbach, P., Stapels, C.J., ... & Norman, E.B. (2003). Low-Energy Coexisting Band in G d 154. *Physical review letters, 91*(10), 102501.

Kulp, W.D., Wood, J.L., Garrett, P.E., Wu, C.Y., Cline, D., Allmond, J.M., ... & Yates, S.W. (2008). Search for intrinsic collective excitations in Sm 152. *Physical Review C—Nuclear Physics, 77*(6), 061301.

Meyer, D.A., Wood, V., Casten, R.F., Fitzpatrick, C.R., Graw, G., Bucurescu, D., ... & Scholl, C. (2006). Extensive investigation of 0+ states in rare earth region nuclei. *Physical Review C—Nuclear Physics, 74*(4), 044309.

Neudatchin, V.G., Struzhko, B.G., & Lebedev, V.M. (2005). Supermultiplet potential model of interaction between the lightest clusters and a unified description of various nuclear reactions. *Physics of Particles and Nuclei, 36*(4), 468-497.

Paes, B., Santagati, G., Vsevolodovna, R. M., Cappuzzello, F., Carbone, D., Cardozo, E. N., ... & Lubian, J. (2017). Long-range versus short-range correlations in the two-neutron transfer reaction $^{64}Ni(^{18}O,^{16}O)^{66}Ni$. *Physical Review C, 96*(4), 044612. https://doi.org/10.1103/PhysRevC.96.044612

Quan, S., Chen, Q., Li, Z.P., Nikšić, T., & Vretenar, D. (2017). Global analysis of quadrupole shape invariants based on covariant energy density functionals. *Physical Review C, 95*(5), 054321. https://doi.org/10.1103/PhysRevC.95.054321

Schwierz, N., Wiedenhover, I., & Volya, A. (2007). Parameterization of the Woods-Saxon potential for shell-model calculations. arXiv preprint arXiv:0709.3525.

Wood, J.L. (2012, December). Do we understand excited 0^+ states in nuclei?. In *Journal of Physics: Conference Series* (Vol. 403, No. 1, p. 012011). IOP Publishing.