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# ACTIVE BOUNDARY LAYER APPROACH TO LAMINAR FLOW UNDER HIGH-PRESSURE AND ENTRANCE CONDITIONS

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**Abstract.** Laminar fluid flow is frequently observed in oilfield operations. In this flow regime, it is crucial to determine the velocity-flow rate performance while considering the fluid's rheological properties for establishing hydraulic criteria. Numerous experiments have validated the laws of resistance and velocity distribution in laminar flow. Nevertheless, some forms of laminar flow require adjustments and additional parameters, particularly at the initial stages, in obliterated laminar flow, or under high-pressure differences. It is important to note that, despite the extensive research on laminar flow regimes, these specific cases or forms of laminar flow have been studied very rarely. This paper investigates the hydrodynamics of specific cases of laminar flow based on the existing active boundary layer model. Calculations were conducted to determine the length of the stable-flow zone (the initial section) of laminar flow. The paper discusses the calculated results. It was determined through calculations that in obliterated laminar flow under high pressure differences, while the fluid flow rate is directly proportional to the pressure difference, the energy loss varies with the square of the pressure difference. Additionally, the effects of pressure and temperature on viscosity and thus flow rates were hydraulically analyzed. The influence of temperature and pressure on flow rate shows an opposite polar effect. In such flow, a small temperature increase does not fully compensate for the changes.

**Keywords**: Laminar flow, velocity profile, pressure loss, hydraulic criteria, viscosity.
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## 1 Introduction

The study of processes occurring in liquids under the influence of pressure and temperature allows us to explain a number of their physical and chemical properties. The study of processes occurring during the storage, transportation and processing of oil and oil products has made fluid physics even more relevant. Therefore, extensive research is being conducted in this direction (Ahmadova, 2020, 2021; Ibragimova et al., 2022; Pashayev et al., 2024). It is well known that laminar flows are one of the most widespread forms of fluid movement. The hydrodynamics of such flows have been extensively studied and investigated (Cherniuk et al., 2024; Kim, 2024). The foundation of the science of viscous fluid motion was established in 1821 by the French scientist Navier and further developed through the work of Stokes. For more than two centuries, models of viscous fluid motion have been based on the assumption of static pressure stability in the cross-sectional area of real fluids. However, recent research has shown that the existing flow model assumes uneven distribution of energetic potential in the cross-section, which does not conform to the thermodynamic stability conditions of the system.

The newly developed active boundary layer model is based not on the stability of static pressure but on the stability of total pressure (the sum of static and dynamic pressures) across the cross-section, with static pressure varying depending on flow velocity (Ismayilov et al., 2024, 2025; Iskandarov et al., 2024, 2025). The changes in parameters across the cross-section according to the new physical flow model are illustrated in Figure 1. This new physical model of flow is critically important for further development in hydro- and gas-dynamics theory, especially for turbulent and multiphase flows, as well as for processes involving mass and heat transfer.

For multiphase flows, considering the new flow model and adapting hydraulic calculations and schemes accordingly is essential to improve the efficiency of transportation processes (Gibeau & Ghaemi, 2022).

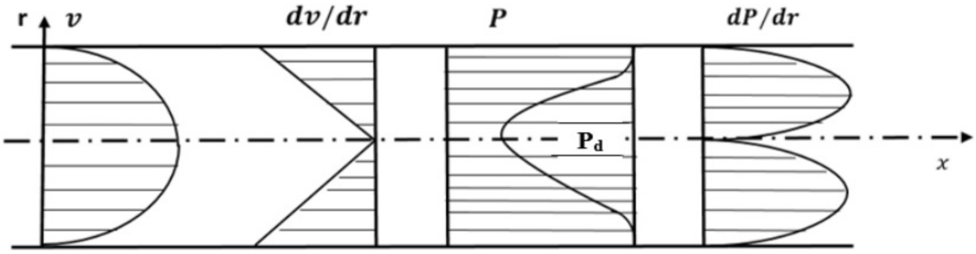


**Figure 1:** Variation of Physical Parameters in the Cross-Section of Cylindrical Flow($v$ - velocity, $dv/dr$ - velocity gradient, $P$ - static pressure, $P_d$ - dynamic pressure, $P_{total}= P + P_d$ - total pressure, $dP/dr$ - pressure gradient)

Laminar flow regimes in multiphase flows are characterized by low stress in the gradient-velocity field. Consequently, there is no cross-sectional transport of matter perpendicular to the flow direction. The stress state of the active medium's boundary layer is defined by the distribution of static pressure ($P$) and shear stress ($t$). Laminar flow is a structured, layered flow pattern of a fluid without mixing, governed by Newton's law of viscosity. The laminar flow of a fluid in a cylindrical pipe can be represented as shown in Figure 2, illustrating the distribution of velocity and shear stress across the cross-section (Robert & Harry, 2003; Martins et al., 2023; Linot et al., 2024).

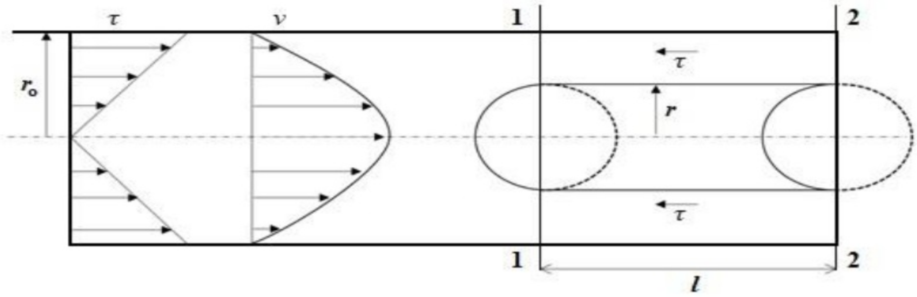


**Figure 2:** Diagram of Laminar Flow of a Fluid in a Cylindrical Pipe

The static pressure of the flowing medium depends on its absolute velocity. Shear stress is determined by the distribution of dynamic pressure across the cross-section of the flow. In laminar flows, the distribution of absolute velocity corresponds to the distribution of the medium's displacement velocity, as there is no displacement velocity across the cross-section of the medium. In turbulent flow, however, the absolute velocity of the medium is equal to the geometric sum of the flow's linear and cross-sectional velocities (Vasques & Beggs, 1980).

Experimental data have shown that in the axis of a turbulent flow, the maximum linear velocity of the medium is 10-20% higher than the average flow velocity ($V_{ave}$). Unlike turbulent flow, in laminar flow, the maximum velocity is twice as high as the average flow velocity, as shown in Figure 3.

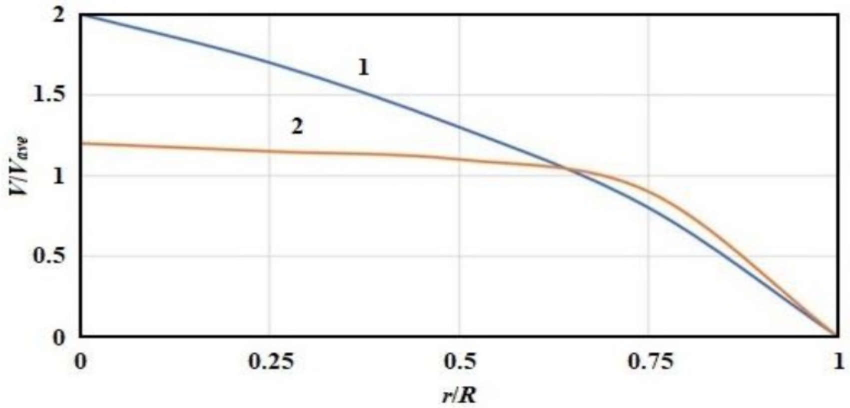


**Figure 3:** Velocity distribution in laminar (1) and turbulent (2) flow.

From hydraulics, it is known that the velocity distribution across the cross-section of the pipe follows a second-order parabolic law during laminar flow, as shown in Figure 3 (Kelbaliyev et al., 2019).

$$v = \frac{\Delta P_{fric}}{4 \times \mu \times l} \times (r_0^2 - r^2) \tag{1}$$

At the center of the flow ($r = 0$), the maximum velocity is:

$$v_{max} = \frac{\Delta P_{fric} \times r_0^2}{4 \times \mu \times l} \tag{2}$$

Here, $l$ is the distance between the (1-1) and (2-2) cross-sections (see Fig.2); $\mu$ is the dynamic viscosity of the flowing fluid; $\Delta P_{fric}$ is the pressure loss (due to friction) occurring over the length $l$; $r_0$ is the radius of the pipe.

Based on the known velocity distribution, the following expressions are also available for determining the flow rate and the average flow velocity:

$$Q = \frac{\Delta P_{fric} \times \pi \times r^{4_0}}{8 \times \mu \times l}, v_{or} = \frac{\Delta P_{fric} \times r_0^2}{8 \times \mu \times l} \tag{3}$$

Based on the last expressions (2) and (3), it can be written that $V_{ave} = V_{max}/2$.

From the expression (3), the pressure loss due to friction in laminar flow can easily be calculated using the well-known Poiseuille-Hagen formula (Saheed & George, 2012; Zehra et al., 2019):

$$H_{fric} = \frac{128 \times \eta \times l \times Q}{\pi \times g \times D^4} \tag{4}$$

Here, $D$ is the diameter of the pipe ($D = 2 \times r_0$); $\eta$ is the kinematic viscosity of the fluid.

By simplifying the last expression (4), we obtain the well-known Darcy-Weisbach equation for calculating the pressure loss due to friction in laminar flows:

$$H_{fric} = \frac{64}{Re} \times \frac{l}{D} \times \frac{v_{ave}^2}{2 \times g} = \lambda_{lam} \times \frac{l}{D} \times \frac{v_{ave}^2}{2 \times g} \tag{5}$$

For laminar flow, despite individual experiments confirming the resistance and velocity distribution laws, adjustments and additions are sometimes necessary for certain flow forms. Let's explore these situations. In laminar pipe flow, at the start where the parabolic velocity distribution gradually forms rather than appearing suddenly, the resistance in this initial section is higher compared to later parts. These effects are generally not considered, except for very short pipelines (Landau & Lifshits, 1992; Kelbaliyev et al., 2019; Vig & Manikantan, 2023).

## 2 Material and Methods

Imagine a fluid entering a pipe with a constant diameter, moving in laminar flow. Initially, the velocity distribution is nearly uniform. Then, due to viscous forces, the velocity gradually spreads across the pipe's cross-section: the fluid layer adhering to the pipe wall slows down, while the central part (core) with uniformly distributed velocity speeds up, allowing a certain flow rate. The thickness of the fluid layer on the wall also increases progressively until it equals the pipe's radius, leading to the characteristic parabolic velocity distribution of laminar flow (see Figure 4).

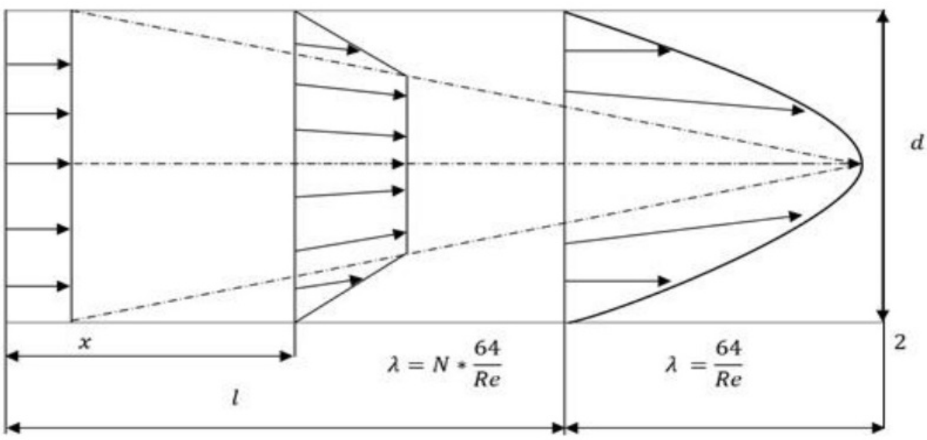


**Figure 4:** Formation of the Parabolic Velocity Profile in Laminar Flows ($l_{b.}$) - beginning of the laminar flow

Thus, the distance up to the point where the parabolic velocity profile characteristic of laminar flow is formed is considered as the special flow-form zone and is referred to as the beginning of the laminar motion zone ($l_{b.}$). The length of this zone can vary depending on the flow velocity and the geometric dimensions of the pipe.

Although the length of the initial zone in laminar flows can be estimated approximately, the following relationship is accepted to determine it:

$$\frac{l_{b.}}{D} = 0.029 \times Re \tag{6}$$

In the last expression, considering the critical value of the Reynolds number for laminar flow ($Re = 2320$), we can obtain the maximum length of the initial zone as:

$$l_{b.}^{max} = 67.4 \times D \tag{7}$$

As seen from the last expression, with the increase in the pipe diameter, the length of the initial laminar flow zone will increase linearly. For a diameter of $D = 1m$, the maximum length of this zone will be 67.4 m. That is, after 67.4 m, a stabilized laminar flow will exist in the pipeline.

Considering the expression in (6), to determine the value of $Re = \frac{v \times D}{\eta}$, we can obtain the following expression:

$$l_{b.} = 0.029 \times v \times D^2/\eta \tag{8}$$

As seen from the last expression, the length of the initial laminar flow zone depends directly on the flow velocity and inversely on the viscosity, with the greatest influence being from the pipe diameter (its square). To evaluate the impact of the mentioned parameters, calculations were made based on equation (10). The calculations were carried out for flow velocities $v = 0.1; 0.3; 0.5; 0.7$ and $1.0\ m/s$ and kinematic viscosities of $5 \times 10^{-7}; 10^{-6}; 5 \times 10^{-6}$; and $10^{-5} m^2/s$, and the dependence of the initial length ($l_beg$) on the pipe diameter ($D$) was determined. The results of the calculations are presented in Table 1, and the dependencies of the $l_{beg} = f(D)$ for laminar flow at velocities $v = 0.1$ and $0.5 m/s$ and 1 m/s are shown in Figures 5, 6 and 7.

**Table 1:** The calculated values for the distance to the stabilization zone for laminar flow.

| Flow velocity,v, m/s | Viscosity of fluid, $\eta$, m$^2$/s | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\eta$=5×10$^{-7}$ | | | | $\eta$=10$^{-6}$ | | | | $\eta$=5×10$^{-6}$ | | | | $\eta$=10$^{-5}$ | | | |
| | Diameter of pipeline D, m | | | | | | | | | | | | | | | |
| | 0,1 | 0,2 | 0,4 | 0,6 | 0,1 | 0,2 | 0,4 | 0,6 | 0,1 | 0,2 | 0,4 | 0,6 | 0,1 | 0,2 | 0,4 | 0,6 |
| 0,1 | 58 | 232 | 928 | 2088 | 29 | 116 | 464 | 1044 | 6 | 23 | 93 | 209 | 3 | 12 | 46 | 104 |
| 0,3 | 174 | 696 | 2784 | 6264 | 87 | 348 | 1390 | 3130 | 17 | 70 | 278 | 626 | 9 | 35 | 139 | 313 |
| 0,5 | 290 | 1160 | 4640 | 10440 | 145 | 580 | 2320 | 5220 | 29 | 116 | 464 | 1044 | 15 | 58 | 232 | 522 |
| 0,7 | 406 | 1624 | 6496 | 12618 | 203 | 812 | 3248 | 7306 | 41 | 162 | 650 | 1461 | 20 | 81 | 325 | 731 |
| 1,0 | 580 | 2320 | 9280 | 20880 | 290 | 1160 | 4640 | 10440 | 58 | 232 | 929 | 2088 | 29 | 116 | 464 | 1044 |

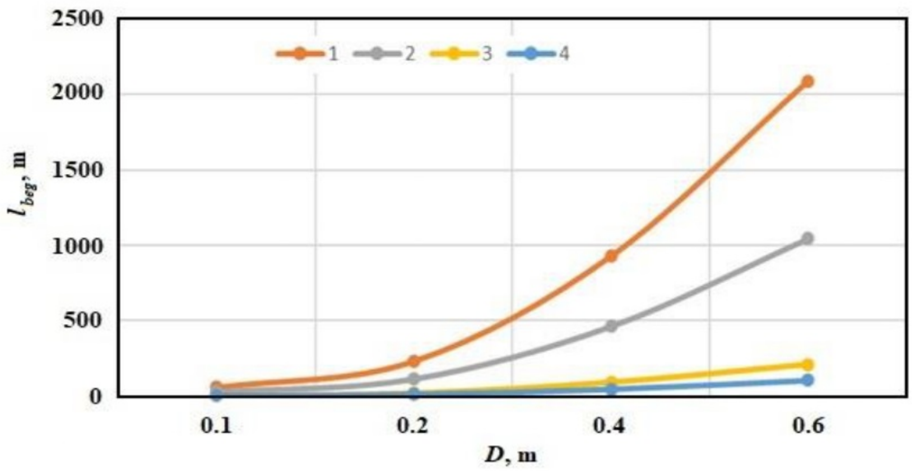


**Figure 5:** The relationship $l_{b.} = f(D)$ ($v = 0.1 m/sec.$) 1-4 respectively, $\eta = 5 \times 10^{-7}$, $10^{-6}, 5 \times 10^{-6}$, $10^{-5}\ m^2/sec.$

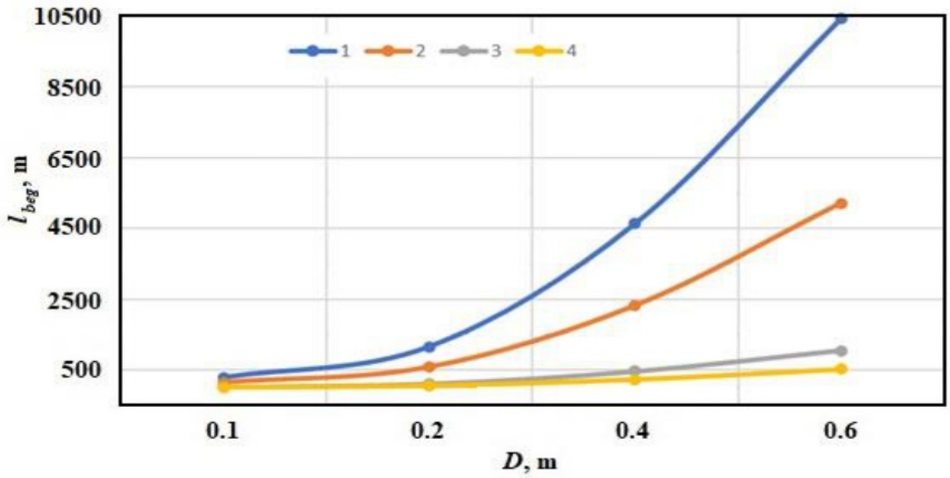


**Figure 6:** The relationship $l_{b.} = f(D)(v = 0,1 = /sec.)$ 1-4 respectively, $\eta = 5 \times 10^{-7}, 10^{-6}$, $5 \times 10^{-6}, 10^{-5} m^2/sec$

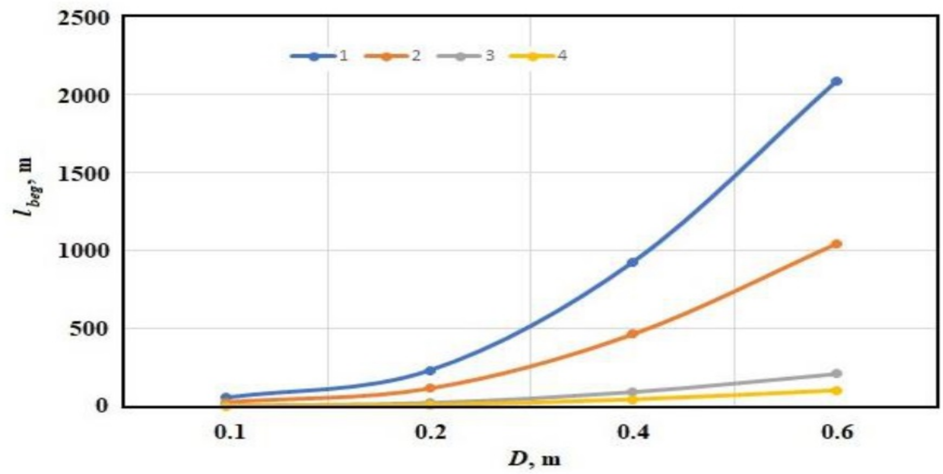


**Figure 7:** The relationship $l_{b.} = f(D)$ $v = 1m/sec$ 1-4 respectively, $\eta = 5 \times 10^{-7}$, $10^{-6}, 5 \times 10^{-6}, 10^{-5} m^2/sec.$

# 3 Results and Discussion

Figures 5, 6 and 7 indicate that the length of the initial zone of laminar flow, i.e., the distance until stabilized flow is achieved, increases quadratically as the diameter of the pipeline grows. As the flow velocity increases, this distance lengthens, while the rise in the kinematic viscosity of the transported fluid proportionally reduces it. In other words, stabilization of the laminar regime occurs over a shorter distance when fluids with high viscosity are transported. From the table, it is clear that the initial distance for laminar flows can span several kilometers, depending on the diameter.

Thus, the analysis shows that resistance at the start of the pipeline is higher compared to subsequent sections. Therefore, when calculating head loss at the start $l < l_{b.}$, the correction coefficient $N$ must be taken into account. This coefficient, greater than one, can be determined according to Schiller's dependency as: $N = \frac{l_{b.}}{D \times Re}$ where $N = 1.09$. This means that at the start, the resistance of the pipeline is 9% higher. Notably, for short pipelines, the $N$ coefficient differs by more than 9% compared to longer pipelines. Considering the starting section, and using expression (4) in conjunction with the Reynolds number formula for laminar flow, we can derive the following equation to calculate pressure loss in laminar flows:

$$P_{fric} = \left(0.165 + \frac{64}{Re} \times \frac{l}{D}\right) \times \frac{v^2 \times \rho}{2 \times g} \tag{9}$$

Here, $\rho$ is the density of the flowing fluid.

From this equation, it is evident that when the relative length of the pipeline ($\frac{l}{D}$) is sufficiently large, the additional term (0.165) in the equation can be ignored. For short pipelines, however, this term must be accounted for. Experiments show that in many practical cases, obliterated laminar flow can also occur. For instance, in laminar flows over short distances or in capillaries under high pressure, i.e., when laminar flow occurs with significant pressure differences, the pressure drop along the flow becomes significantly nonlinear and deviates sharply from Poiseuille's law. This is explained by the fact that although the fluid discharge in laminar flow is proportional to the pressure difference, the energy lost is proportional to the square of the pressure difference. As a result, energy loss per unit discharge increases with the pressure difference. Consequently, at high pressure differences, fluid heating occurs, leading to a decrease in viscosity. On the other hand, the pressure drop along the flow reduces the fluid's viscosity compared to its value at the start. Thus, viscosity becomes a variable parameter along the flow.

Viscosity decreases due to two factors. In contrast, for flow rate, an increase in temperature causes an increase, while a rise in pressure results in a decrease. Therefore, these two factors have opposite effects on flow rate. It is worth noting that full compensation does not occur due to heat transfer from the pipe walls (resulting in only a slight temperature rise).

Such flows occur in high-pressure hydraulic machines and capillaries under significant pressure differences. Below, the hydrodynamic analysis of such a flow is considered. Assume that laminar flow occurs between two parallel planes (in a narrow gap). The task is to investigate the effect of changes in viscosity caused by variations in pressure and temperature on the flow. Let us assume that the fluid's density does not change with pressure and temperature, and $a/b \to 0$ (Figure 8).

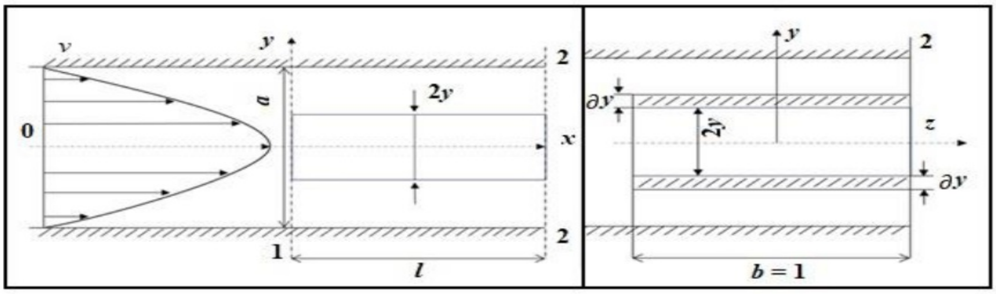


**Figure 8:** Schematic of laminar flow in narrow gaps

The distance between the plates is $a$, and the origin of the coordinates is considered at the center of the gap. The distance between the two cross-sections is $l$, and the width of the flow is taken as 1 (unit). Thus, we are looking at the volume of the fluid in the shape of a rectangular parallelepiped. This volume is $l \times 2y \times b$, (where $b = 1$). For the steady motion of the fluid along the x-axis, the following condition can be written:

$$2y \times b \times \Delta P = -\mu \times \frac{\partial v}{\partial y} \times 2l \times b \tag{10}$$

Here, $\Delta P$ is the pressure difference (in the cross-section).

From the last expression, considering v=0 when $y = \frac{a}{2}$, the following equation for velocity distribution can be easily obtained:

$$v = \frac{\Delta P}{2\mu} \times \left(\frac{a^2}{4} - y^2\right) \tag{11}$$

Then the elemental flow rate per unit width ($q$) would be as follows:

$$dq = v \times dS = \frac{\Delta P}{2\mu} \times \left(\frac{a^2}{4} - y^2\right) \times 2dy = \frac{\Delta P \times a^3}{12\mu} \tag{12}$$

Considering that the total flow rate $Q = q \times b(b \neq 1)$, the final expression for calculating the pressure loss is:

$$\Delta P = \frac{12 \times \mu \times Q}{a^3 \times b} \tag{13}$$

Now, to evaluate the effect of temperature on viscosity, we can write the energy equation based on the equality between the energy loss due to friction (converted to heat) and the increase in the thermal energy of the fluid per unit time:

$$Q \times \rho \times c \times (t - t_1) = k \times (P_1 - P) \times Q \tag{14}$$

From the last equation, we obtain:

$$(t - t_1) = \frac{k}{\rho \times c} \times (P_1 - P) \tag{15}$$

Here, $c$ is the heat capacity of the fluid ( kJ/kg·°C),

$P$ is the pressure (Pa),

$k$ is the proportion of the work done by the viscous forces for heating the fluid.

If $k = 1$, there is no heat transfer from the pipe wall, so all the work done by the viscous forces will be used to heat the fluid.

If $k = 0$, there is intensive heat transfer from the pipe wall, so the fluid's temperature does not increase (isothermal flow).

To consider the joint effect of pressure and temperature on viscosity, the following known expression can be used:

$$\mu = \mu \times e^{[\ a \times (P - P_1) - \lambda \times (t - t_1)]} \tag{16}$$

Considering equation (15) and (16), we can obtain the following expression for determining viscosity:

$$\mu = \mu_1 \times e^{\left[(P - P_1) \times \left(\alpha + \frac{k \times \lambda}{\rho \times c}\right)\right]} \tag{17}$$

Then, the following expression can a lso be written:

$$\frac{12 \times Q \times \mu_1}{a^3} dx = -\int e^{\left[(P - P_1) \times \left(\alpha + \frac{k \times \lambda}{\rho \times c}\right)\right]} \times dP$$

If we integrate the last expression, we will obtain:

$$\frac{12 \times Q \times \mu_1 \times x^3}{a} dx = -\frac{1}{\alpha + \frac{k \times \lambda}{\rho \times c}} \times\ e^{\left[(P - P_1) \times \left(\alpha + \frac{k \times \lambda}{\rho \times c}\right)\right]} + C$$

When $x = 0$, we have $P = P_1$, so:

$$C = -\frac{1}{\alpha + \frac{k \times \lambda}{\rho \times c}}$$

When $x = 1$ , $P = 0$, the result is:

$$Q = \frac{a^3}{12\mu_1 \times l} \times \frac{1}{\alpha + \frac{k\times\lambda}{\rho\times c}} \times \left[ e^{\left[P_1 \times \left(\alpha + \frac{k\times\lambda}{\rho\times c}\right)\right]} - 1 \right] \tag{18}$$

Here, $\mu_1$ is the viscosity of the fluid at the beginning of the flow (when $P = P_1$ and $t = t_0$). This viscosity can also be expressed as $\mu_0$:

$$\mu_1 = \mu_0 \times e^{a\times P} \tag{19}$$

In the special case, if the flow is isometric, then the equation (19) for fluid flow rate will be as follows:

$$Q = \frac{a^3}{12\mu_0 \times l \times \alpha} \times (1 - e^{-\alpha\times P_1}) \tag{20}$$

To determine the relative flow rate ($\overline{Q}$), we divide the expression (18) for $Q_0$ (when $\mu = \mu_0 = const$) by the given expression. This gives us:

$$\overline{Q} = \frac{Q}{Q_0} = \frac{e^{-\alpha\times P_1}}{P_1 \times (\alpha + \frac{k\times l}{\rho\times c})} \times \left[ e^{\left[P_1 \times \left(\alpha + \frac{k\times\lambda}{\rho\times c}\right)\right] - 1} \right] \tag{21}$$

Based on the last equation, calculations were carried out for various pressures ($P_1$) (from 0 to 800 $kgf/cm^2$), $\alpha = 1/430\ \frac{cm^2}{kgf}$, $\lambda = 1/36\ \frac{1}{^\circ C}$, $c = 2.1 \frac{kJ}{kq\times {}^\circ C}$, $\rho = 850 \frac{kg}{m^3}$. As a result, for cases of $k = 1$ and $k = 0, \overline{Q} = f(P_1)$ dependencies were developed (See Fig.9).

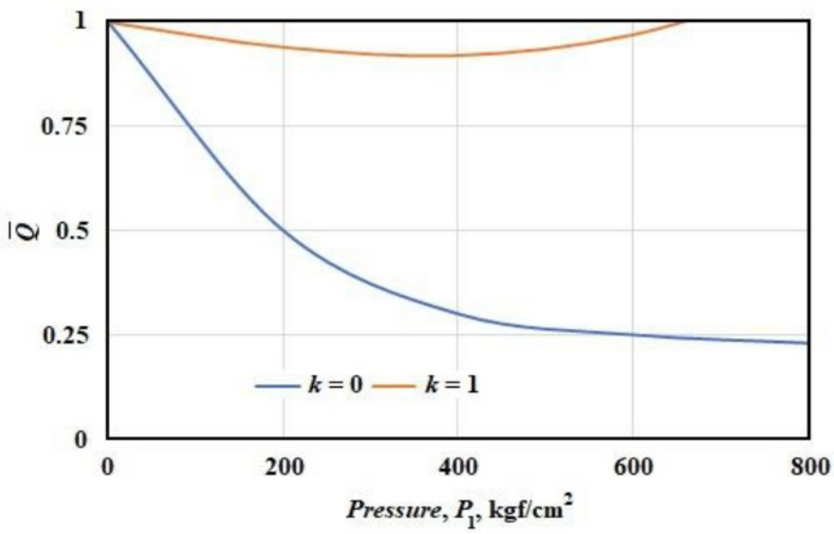


**Figure 9:** The effect of pressure (viscosity) change on the flow rate $k = 1$- no heat transfer occurs to the outside; $k = 0$ - for the isothermal process.

In Figure 9, $k = 1$ represents the case where there is no heat transfer from the pipe wall, while $k = 0$ characterizes the isothermal condition. As seen from Figure 9, the most likely flow regime corresponds to the case where $k = 0$, where the heat change plays a maximal role.

# 4 Conclusion

1. According to the active boundary layer model, the initial zone, obliterated and high-pressure flows, which are considered special cases of laminar flows, were hydrodynamically investigated.

2. The issue of hydraulic investigation and determination of the length of the zone up to the formation of the parabolic velocity profile in laminar flows was considered.

3. In obliterated laminar flows that occur at a high pressure difference, although the fluid flow rate is proportional to the pressure difference, the lost energy changes proportionally to the square of the pressure difference. The effect of temperature and pressure factors on flow rate is manifested as an opposite polar effect. In such flows, due to a small increase in temperature, full compensation does not occur.

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