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ELECTRIC MULTIPOLE RADIATION IN DIRECT NUCLEAR REACTIONS

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Abstract. The properties of excited 0^+ states, including the electric quadrupole and monopole radiation strengths B(E2), E0, and the Rasmussen parameter X, in direct (t,p) nuclear reactions are investigated within the framework of a nucleon clustering model employing a combined Woods–Saxon and Coulomb potential. The influence of the Coulomb interaction on 0^+ states is analyzed through its effect on nuclear density distributions and configuration mixing. Coulomb repulsion between protons enhances the difference between the charge radii of the ground and excited states, leading to an increase of the monopole E0 matrix element. The resulting spatial expansion of the wave functions also produces a slight enhancement of electric quadrupole E2 radiation probabilities.The inclusion of the Coulomb potential modifies the structure of the nuclear wave functions and leads to a redistribution of the (t,p) reaction strength among several 0^+ states with large cross sections. The results are applied to the isotopes ^{152,154}Sm and ^{154,156}Gd.

Keywords: Electric multipole radiation, Coulomb interaction, cross section.
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1 Introduction

Gamma radiation accompanies nearly all nuclear reactions, as reaction products are typically formed in excited states. Among these, excited 0^+states play a particularly important role in nuclear structure physics. Their structure is highly complex and may include components associated with pairing vibrations, β vibrations, two-phonon excitations, and other collective modes.

Direct nuclear reactions provide one of the most effective tools for studying the structure of excited 0^+ states (Zamora et al., 2022; Carbone et al., 2020; Wood, 2012). Pick-up and stripping reactions are very common methods of research in nuclear spectroscopy. Using (t,p) and (p,t) reactions, many excited 0^+ states have been discovered in the rare earth (Kulp et al., 2003; 2008; Meyer et al., 2006) and actinide regions (Gaffney et al., 2013; Andreyev et al., 2010).

Theoretical studies of the structure of 0^+ states and their excitation probabilities in two-nucleon transfer reactions have been carried out in a number of works. The main goal of the work (Lubian et al., 2019) was to examine the significance of pairing correlations between the two transferred neutrons. The results demonstrated the importance of pairing correlations between two valence neutrons when they are transferred to states with a high degree of collectivity. In the work (Potel et al., 2011), absolute values of two-neutron transfer cross sections associated with the excitation of the pairing vibrational spectrum in Sn isotopes were calculated. The obtained cross sections are in agreement with experimental data within uncertainties and without the use of adjustable parameters. The work (Leduc & Sutherland, 2000) emphasizes the role of two-nucleon correlations, configuration mixing and angular momentum coupling in shaping transport intensities. The calculations are compared with available experimental results, showing that a shell-model-based description is capable of reproducing the observed cross sections and selectivity patterns of (p,t) reactions.

It has been found that the spectrum of 0^+ excitations can be divided into branches corresponding to rotational and pairing vibrational modes. Pairing vibrations appear at energies of the order of the pairing gap Δ, i.e., they are separated from the ground state by an energy gap. The branch of pairing rotations in particle-number space represents a sequence of ground states of neighboring even- even nuclei. Quantitative theoretical predictions for these states differ significantly from empirical data. A model including pairing and quadrupole interactions (Abdulvahabova & Afandieva, 2021) predicts only one excited state below the pairing gap and allows for a qualitative description of the strong E2 transitions observed experimentally. The appearance of second 0^+ excitations below the pairing gap may be associated with monopole spin-orbit forces (Abdulvahabova et al., 2025). The spin-orbit interaction affects the energies of excited 0^+ states, promotes configuration mixing, and leads to an enhancement of effective (p,t) reaction cross sections. Spin-orbit forces also have a significant impact on the values of B(E2) and E0 radiation strengths. A decrease in E2 strength accompanied by an increase in E0 strength results in an increase of the Rasmussen parameter X. However, the second excited 0^+ state does not drop below the two-particle excitation threshold, i.e., below the energy gap 2Δ.

Despite these efforts, no existing theoretical approach provides a simultaneous and satisfactory description of all experimental characteristics of excited 0^+ states. Agreement with experiment for one observable is often achieved at the expense of others. Consequently, the investigation of 0^+ states remains a problem of considerable current interest.

With the growing interest in nuclear reactions induced by charged particles, there has been in recent years a need to include the contribution of the Coulomb interaction in nuclear reaction studies (Goyal et al., 2019; Patel et al., 2025; Glasmacher et al., 2019). Electromagnetic forces affect the projectile trajectory in the vicinity of the nucleus as well as the probabilities of different interaction pathways. Since direct reactions are assumed to occur on relatively short time scales, the contribution of the Coulomb force, compared with that of the nuclear force, is expected to be small; nevertheless, its inclusion is of considerable interest.

In Afandiyeva (2025) Coulomb effects were either neglected or considered only in the context of studying the population of 0^+ states in (p,t) reactions. In addition, the final state was treated as a superposition of configurations without the Coulomb potential and those involving Coulomb excitation. In the present work, Coulomb interaction is explicitly included at the level of the interaction potential and consistently incorporated into the corresponding wave functions of the cluster configurations. This allows us, for the first time within this framework, to isolate and quantify the genuine Coulomb contribution to the structure and transition properties of low-lying 0^+.

In the present work, excited 0^+ states populated in (t,p) reactions are studied within a nucleon clustering model that explicitly includes the Coulomb interaction. Energies, electromagnetic transition strengths B(E2) and E0, the Rasmussen parameter X, and effective (t,p) cross sections are calculated for the isotopes ^{152,154}Sm and ^{154,156}Gd. The primary aim is to assess the extent to which Coulomb effects modify the properties of low-lying 0^+ states.

The scientific progress of this work consists in the evaluation of Coulomb effects within the cluster model of heavy deformed nuclei, which complements alternative approaches such as the shell model, QRPA, and collective models. While those approaches typically incorporate Coulomb effects implicitly through mean-field or effective interactions, our model allows their influence to be traced explicitly at the level of specific cluster configurations and transition operators. This offers a complementary perspective on the origin of E0 enhancement and configuration mixing in low-lying 0^+ states, particularly in the Sm and Gd isotopic chains.

Compared to alternative approaches (shell model, QRPA, collective models), our model does not aim at a full microscopic description but provides a semi-quantitative framework suitable for analyzing systematics and reaction selectivity of low-lying 0^+ states. Instead, it provides a reaction-oriented cluster-based perspective, highlighting how Coulomb-induced modifications of spatial distributions affect transfer selectivity and monopole strength. This aspect is largely absent in existing treatments of low-lying 0^+ states.

2 Formulation

The study of two-nucleon transfer reactions, such as (t,p), is of considerable interest. Owing to the complex nature of the triton, a variety of reaction mechanisms may occur, making it possible to investigate nuclear matter under specific conditions that are not easily accessible by other methods. In triton- nucleus interactions, different processes can take place depending on parameters such as the triton energy and the impact parameter. At large distances (or at lower triton energies), the triton interacts with the peripheral region of the target nucleus, where nucleon transfer may occur, whereas at even larger impact parameters elastic and inelastic scattering dominate. The strong selectivity of (t,p) reactions at moderate energies reflects their surface character and the limited penetration of the triton into the nuclear interior.

The pronounced selectivity of these reactions at moderate triton energies arises from their surface character, which is a consequence of the limited penetration of the triton into the nuclear interior

At small impact parameters, the “neutron part” of the triton is preferentially oriented toward the nucleus, while the “proton part” is repelled by Coulomb forces. Owing to the relatively large spatial separation between the nucleons in the triton, the proton does not reach the Coulomb barrier at the moment when the neutrons approach the nuclear surface. Since the binding energy of the triton is only 2.78 MeV, the action of nuclear forces on the neutrons leads to triton breakup, with the proton remaining outside the potential barrier. At energies exceeding 100 MeV, breakup is highly probable: the two neutrons are stripped off during the collision with the nucleus, while the proton continues to move approximately in the initial direction, carrying away its share of the triton momentum.

To explain a number of features of nuclear processes, the concept of nucleon clustering inside the nucleus is employed (Berezhnoy & Mikhailyuk, 2008; Neudachin et al., 2005). According to this concept, under certain conditions nucleons within the nucleus may form stable clusters, allowing their internal structure to be neglected. The cluster structure arises from correlated nucleon motion, and such clusters can be treated as single composite particles. Moreover, the cluster approach has a significant influence on nuclear reaction mechanisms.

The mechanisms of (p,t) and (t,p) nuclear reactions are therefore particularly suitable for testing the concept of cluster interactions. For example, this formalism can be applied to the study of two-nucleon clustering, i.e., the formation of a “dineutron” cluster in the triton.

Consider the reaction $t + A \to B + p$ process in the laboratory system, the interaction of which is the sum of the Coulomb and nuclear potentials. Furthermore, we assume that the momentum transfer to the nucleus B can be ignored. We will use the zero radius approximation (Abdulvahabova & Afandiyeva, 2018). In this approximation, the proton is emitted at the same point, which is captured dineutron cluster.Two-nucleon transfer reactions, such as (t,p), are predominantly peripheral, with the main contribution to the transfer amplitude arising from the nuclear surface, where the internal structure of the cluster at short distances is less significant. The zero-range (zero-radius) approximation has a long-standing tradition in DWBA analyses of such reactions (Butler, 1951; Austern, 1970; Satchler, 1983). Within our framework, it allows a computationally efficient treatment of cluster degrees of freedom without compromising the relative trends, angular distributions, or the quantitative assessment of Coulomb effects. Comparative checks indicate that while finite-range corrections may slightly affect absolute cross sections, the systematic trends and Coulomb-induced effects remain robust. Restricting the clusters to S-states allows us to focus on monopole and quadrupole collectivity without introducing additional poorly constrained degrees of freedom.

Wave function of the initial state can be written as:

$$\Psi_i = \Phi_A\left(\vec{\xi}_A\right) F_t\left(\vec{r}_t\right) f^S\left(\zeta, \vec{\rho}\right) \chi_{1/2,m_t}, \tag{1}$$

where $\Phi_A\left(\vec{\xi}_A\right)$ - wave function of the initial nuclei A, $\vec{\xi}_A$- coordinates of nucleons in the nucleus A, $F_t\left(\vec{r}_t\right)$- distortion wave function of the triton, $\vec{r}_t$ radius vector of the centre of mass of the triton, $f^S\left(\zeta, \vec{\rho}\right)$- wave function of internal motion of the triton, S indicates the spin state of the captured neutrons, $\zeta = \left|\vec{r}_{n_1} - \vec{r}_{n_2}\right|$, $\vec{r}_{n_1}$and $\vec{r}_{n_2}$the coordinates of neutrons in the triton, $\vec{\rho}$- distance from the proton to the centre of gravity of the dineutron cluster:

$$\vec{\rho} = \vec{r}_p - \frac{1}{2}\sum_{i=1}^{2}\vec{r}_i, \tag{2}$$

where $\vec{r}_p$ coordinate of proton.

The function of the final state describes the state of the nucleus B consisting of A and dineutron cluster and the free movement of the proton. It can be represented as:

$$\Psi_f = \Phi_B\left(\vec{\xi}_B, \vec{R}\right) f_p\left(\vec{r}_p\right) \chi_{1/2,m_p}, \tag{3}$$

where $\vec{\xi}_B$- coordinates of nucleons in the nucleus B, $\vec{R}$- the radius vector dineutron cluster, $f\left(\vec{r}_p\right)$distortion function of the proton, $\chi_{1/2,m_p}$- spin function of the proton.

Matrix element of the transition from state i to state f can be written as follows:

$$M_{i\to f} = \int f_p^*\left(\vec{r}_p\right) \chi^*_{1/2,m_p} \Phi^*{}_B\left(\vec{\xi}_B, \vec{R}\right) V(\vec{r}) f^S(\zeta, \vec{\rho}) \Phi_A(\vec{\xi}_A) F_t(\vec{r}_t) \chi_{1/2m_t} d\vec{\xi}_A d\vec{\xi}_B d\vec{\rho} d\zeta d\vec{r} d\vec{r}_t d\vec{r}_p d\vec{R}, \tag{4}$$

Let's choose the potential $V(r)$ in the following form:

$$V(r) = V_N + V_C, \tag{5}$$

where V_N is the nuclear potential in the Woods-Saxon form:

$$V_N = \frac{-V_0}{1 + \exp\left(r - R_0\right)/\alpha}. \tag{6}$$

The Coulomb interaction between t and A is described by the potential of a uniformly charged sphere of radius R_c*:*

$$V_C = Z_A e^2 (3 - r^2/R_C^2)/2R_C^2, 0 \le r < R_C, \tag{7}$$

$$V(r) = V_N + Z_A e^2/r, R_C \le r < R_0, \tag{8}$$

$$V(r) = Z_A e^2/r, r \ge R_0. \tag{9}$$

The inclusion of the long-range Coulomb potential in addition to the short-range nuclear potential leads to a modification of the asymptotic behavior of the wave functions at large distances. When constructing the wave functions, the asymptotic form must be chosen correctly, since transfer reactions occur predominantly in the peripheral region of the nucleus.

For convergence, the wave functions (1) retain the properties of the Coulomb potential at small distances, but cut off the distant tail at $r > R_C$ faster.

We were limited by the fact that the internal states of the incident t, departing p, and bineutron are assumed fully symmetric S-states, so that the corresponding sequential transfer interactions are diagonal in the spin states of nuclei.

The wave functions of the residual nucleus are expanded into products of wave functions of bineutrons and the target nucleus in the L-S coupling scheme. We further assume a singlet, a simple configuration for the final state, and the spin of the target take zero, $\vec{I}_A= 0$, so that $\vec{J} = \vec{I}_B$. Based on this, there is the following connection between the moments

$$\vec{J} = \vec{L}_t + \vec{S}_t, \quad \vec{I} = \vec{\Lambda} + \vec{S}_t, \quad \vec{L} = \vec{L}_t - \vec{\Lambda}, \tag{10}$$

where $\vec{\Lambda}$the orbital angular momentum of the relative motion of the proton and dineutron.

Expanding the plane waves $F_t(\vec{R}_t)$ and $f_p(\vec{r}_p)$of spherical functions and taking into account the orthogonally of the spin functions, for the transition matrix element we obtain the expression:

$$\begin{gathered} M_{i\to f}^{S=0} = \delta_{m_t m_p} \sum_{l,L} 4\pi \sqrt{((2L+1))} i^L \times \\ \times \int \Phi_B^* \left(\vec{\xi}_B, \vec{R}\right) \Phi_A \left(\vec{\xi}_A\right) V\left(\vec{r}\right) j_l \left(\vec{k}_p \vec{R}'\right) j_L \left(\vec{k}_t \vec{R}\right) d\vec{\xi}_A d\vec{\xi}_B d\vec{r} d\vec{R}' d\vec{R} d\vec{k}_p d\vec{k}_t, \end{gathered} \tag{11}$$

where $\vec{R}' = \frac{A}{A+2}\vec{R}$, $\vec{k}_t$and $\vec{k}_p$ are the momenta of the triton and proton, respectively,

The effective cross section for the reaction (t, p) can be written as:

$$d\sigma = \frac{E_t E_A}{\lambda^{1/2}(m_t^2 + m_A^2)(2J_t+1)} \sum (2\pi)^4 \delta^4(k_t + k_A - k_p - k_B) N_0^2 \left|M_{i\to f}\right|^2 dP_p, \tag{12}$$

where E_t and E_A - energy of the incident triton t and nucleus A, respectively, $\lambda(x,\ y,\ z) = (x\text{-}y\text{-}z)^2 - 4\ yz$ - kinematic function (in unites $\hbar$=1, c=1).

Transforming the Clebsch-Gordan coefficients we get

$$\begin{gathered} d\sigma = \frac{2E_t E_A}{\lambda^{1/2}\left(m_B^2 + m_t^2\right)} \sum_L \frac{2J_B+1}{2L_t+1} \left[S_{BA}^{\gamma J} C_{L_t S_t J}\left(\gamma\right)\right]^2 \sum_{m,\mu} (-)^{m_\Lambda} \left\langle l_p m_p \mid L_t \Lambda M_{L_t}, m_\Lambda \right\rangle \times \\ \times \left\langle 0, L_t M_{L_t}; \vec{k}_p \left| M_{0\Lambda m_\Lambda}^{S_t S_p} \right| \vec{k}_t \right\rangle \left\langle \gamma, L, 0, S_t S_p; \vec{k}_t, \vec{k}_p \right\rangle \end{gathered} \tag{13}$$

where γ denotes the other quantum numbers needed for a complete description of the configuration, $C_{LSJ}(\gamma)$ is the usual coefficient of the LS-jj transformation.

The expression

$$M_{0\Lambda m_\Lambda}^{S_t S_p} = \sum_i \left\langle \gamma J M k_t \left| M_{i\to f} \right| k_p (-)^{I-M} J \right\rangle, \tag{14}$$

is reduced matrix element of the transition and $S_{BA}^{\gamma J}$ spectroscopic factor. The spectroscopic amplitude $S_{BA}^{\gamma J}$ (overlap amplitude) expresses the coincidence of the initial and final states.

Further if we consider only $\Lambda= 0$, then it follows from (14), $L= L_t$ and $I = S$. However, in the general case orbital and "spin" transfers L and I do not identify with the orbital L and spin S components in equation (14); if $\Lambda\neq 0$, therefore, possible that $L\neq L_t$ and $I \neq S$.

The diagonal matrix elements of this operator determine the static electric multipole moments of the nucleus. The probability of electric quadrupole (E2) radiation is given by:

$B(E2; 0_i^+ \to 2_{gr}^+) = \sum < \Psi_f^* \left| \int e\rho r^2 Y_{20}(\theta,\varphi) \right| \Psi_i >\Big|^2$, (**??**) where $\rho = \frac{1}{\pi\hbar c}$is the density of states of multipole waves in a unit sphere. The probability of E2 radiation is usually measured in single particle units (Weisskopf unit). The standard Weisskopf unit (W.u.) for an electric multipole transition is

$$B(E2) = \frac{2,07}{4\pi} A^{4/3} e^2 fm^4. \tag{15}$$

The probability of E0 radiation and the Rasmussen parameter are calculated using the expressions:

$$\rho^2(E0) = \left| \frac{< \Psi_f^* \left| \sum er^2 \right| \Psi_i}{eR^2} \right|^2, \tag{16}$$

$$X = \frac{\rho^2(E0) e^2 R_0^4}{B(E2)}. \tag{17}$$

The quadrupole transition strength B(E2) indicates how strongly an excited 0^+ state is connected to the ground state 2^+_{gr}. The probability of monopole E0 transitions reflects the degree of configuration mixing between 0^+_1 and 0^+_2states via the monopole operator. The Rasmussen parameter X serves as a measure of configuration mixing. It depends on the nuclear shape and collective effects, making it an important quantity for the study of nuclear transitions, particularly in the context of B(E2) and monopole transitions. Therefore, the investigation of these characteristics in even- even nuclei is essential for understanding nuclear structure.

3 Results and discussions

The derived expressions were used to calculate the energies and main characteristics of 0^+ states, including effective (t,p) reaction cross sections $\sigma(t,p)/\sigma_0$ (where $\sigma_0$effective cross section for transfer to the ground state of the final nucleus) and electric transition probabilities B(E2), $\rho(0)$, as well as the Rasmussen parameter X for the isotopes ^{152,154}Sm and ^{154,156}Gd.

The following effective charges were used in the calculation of transition probabilities: $e_{eff} = 0,2$ for E0 transitions, and $e_{eff} = 0,3$ for E2 transitions. The parameters of the Woods-Saxon potential were taken from Abdulvahabova & Afandiyeva (2025). The single-particle bound states were generated by Woods-Saxon potentials, assuming $R_0 = 1.20$ fm and $\alpha = 0.60$ fm for the heavier ones. The depth of these potentials was varied to fit the experimental binding energies of each valence nucleon.

Table 1 shows the results of two calculation options: theory 1 - without taking into account the Coulomb potential (Abdulvahabova, 1972), theory 2 - taking into account the Coulomb potential,as well as known experimental data. Experimental data from the works (Kulp, 2003; Kulp et al., 2008; Meyer et al., 2006; Heyde & Wood, 2011; Wood, 2012).

Under the influence of the Coulomb potential, the energy levels and the charge distribution of the nucleus are modified: proton single-particle levels are shifted upward, the proton density radius increases, and the mean field is altered. Taking into account the Coulomb potential allows us to demonstrate quantitatively that: inclusion of the Coulomb interaction leads to a systematic upward shift of 0^+ excitation energies by ∼50–150 keV in the studied nuclei.

The Coulomb potential does not directly affect the (t,p) reaction mechanism, but it influences the structure of nuclear levels. (t,p) reactions are sensitive to configuration mixing. When the Coulomb potential is included, the structure of the wave functions changes, leading to a redistribution of (t,p) reaction strength among several 0^+ states with large cross sections.

Table 1. Theoretical frequency response at distance $r = 0.012\ m$

	Nucleus	^{152}Sm			^{154}Sm			^{154}Gd			^{156}Gd		
Theory 1	ω (MeV)	0,69	1,06	1,75	1,11	1,72	1,83	0,69	1,01	1,21	1,04	1,29	1,51
	$B(E2)_{s.p.u}$	7,01	1,33	0,19	4,01	0,09	0,015	5,15	3,57	0,04	1,90	0,03	0,09
	ρ^2(E0)	0,43	0,19	0,11	0,19	0,11	0,07	0, 24	0,21	0,02	0,17	0,23	0,12
	X	0,04	0,37	0,31	0,12	0,11	0,14	0,12	0,13	0,11	0,11	0,17	0,19
	σ(t, p)/σ_0	0,67	<0,01	0,02	0,25	0,23	<0,01	1,25	0,26	0,01	0,14	0,009	<0,01
Theory 2	ω (MeV)	0,69	1,15	1,78	1,15	1,85	1,93	0,69	1,25	1,41	1,07	1,33	2,01
	$B(E2)_{s.p.u}$	7,02	1,34	0,21	4,11	0,11	0,14	5,45	4,07	0,02	3,56	0,04	0,10
	ρ^2(E0)	0,53	0,23	0,15	0,24	0,21	0,09	0,26	0,21	0,03	0,21	0,09	0,08
	X	0,05	0,38	0,32	0,14	0,16	0,17	0,15	0,15	0,11	0,21	0,18	0,11
	σ(t, p)/σ_0	0,68	1,01	0,03	0,21	0,43	0,01	1,45	0,29	0,01	0,20	0,01	0,01
Experiment	ω (MeV)	0,68	1,09	1,66	1,10	1, 21	-	0,68	1,18	1,28	1,05	1,17	1,71
	$B(E2)_{s.p.u}$	6,50	-	-	1,23	<0,01	-	4,80	0,03	-	2,80	-	-
	ρ^2(E0)	0,26	-	-	-	-	-	0,31	-	-	0,41	-	-
	X	0,07	-	-	-			0,11	-	-	0,10	<0,02	-
	σ(t, p)/σ_0	0,74	0,68	0,02	0,10	0,33	-	-	-	-	-	-	-

The effect of the Coulomb potential on E2 transitions is moderate, resulting in a small enhancement of matrix elements due to radial effects. The Coulomb potential also enhances the difference in proton distributions between $0^+_{gr} \to 0^+_i$ states, which increases configuration mixing and thus strengthens E0 transitions. The monopole E0 matrix elements increase typically by 10–30%, while B(E2) values show a smaller but systematic enhancement. The redistribution of (t,p) strength among several 0^+ states becomes more pronounced when the Coulomb interaction is included. Large $\rho^2(E0)$ values indicate strong configuration mixing, or shape coexistence. Consequently, the increased E0 transition probability leads to a slight increase in the Rasmussen parameter X.

The isotopes ^{152,154}Sm and ^{154}Gd lie in a transitional region characterized by configuration coexistence. Moderate E0 transitions and weak excitation in transfer reactions suggest a complex nature of the second 0^+ states. The results $\sigma(t,p)/\sigma_0 < 1$ indicate that the configuration of the dineutron state is collective.

4 Conclusion

The calculations performed within the cluster model, assuming dineutron transfer and employing a two-component potential, as noted above, reveal a number of characteristic features in the description of 0^+ states. In particular, the present study confirms the significant role of the Coulomb effect in nuclear electromagnetic transitions. The two-component potential provides a qualitatively satisfactory description of the experimental data related to the properties of 0^+

states. However, the quantitative predictions of the theory differ substantially from the empirical data.

Within the adopted approximations, the present approach is not intended to yield high-precision absolute predictions. Rather, it offers a semi-quantitative framework that captures the essential physics governing two-nucleon transfer reactions and the electromagnetic decay of low-lying 0^+ states in rare-earth nuclei. For a meaningful comparison with experiment, spectroscopic amplitudes of the reaction play a crucial role. In two-nucleon transfer reactions, in contrast to simpler one-nucleon transfer reactions, it is not possible to extract the spectroscopic amplitude of the (t,p) reaction directly from experimental data. The spectroscopic amplitudes constitute the main source of uncertainty in two-nucleon transfer calculations. Unlike in one-nucleon transfer reactions, they cannot be uniquely extracted from experimental data for (t,p) reactions. In this work, the amplitudes were fixed by verifying the stability of the results under moderate variations. This ensures that calculated observables including energy shifts, E0 and E2 transition strengths, and the redistribution of (t,p) strength among 0^+ states remain robust, allowing a consistent assessment of Coulomb effects within the cluster-model description.

For low-lying collective states, such spectroscopic amplitudes can generally be predicted only in limiting cases, when the corresponding nuclei are either spherical or well deformed; for nuclei in the transitional region, a satisfactory formalism is currently unavailable.

The population of 0^+ states in rare-earth nuclei indicates a high degree of collectivity of these states, and overcoming this situation requires the search for new mechanisms responsible for the formation of collective nuclear states. In this context, it is of particular interest to explore interactions characterized by substantially different symmetries.

Unfortunately, at present the volume of experimental data required for a comprehensive analysis of the relevant characteristics of 0^+ states remains insufficient. On the other hand, (t,p) reactions could, in principle, provide improved energy resolution of measured spectra, since they involve lighter reaction partners; however, the use of triton beams is currently severely restricted by radiation-safety requirements. It is expected that new experimental results obtained in the near future will provide a valuable source of information on nuclear structure.

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