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ALGORITHMIC APPROACH FOR THE MATHEMATICAL REPRESENTATION OF THE COLLATZ PROCEDURE

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Abstract. The Collatz Conjecture goes beyond being merely a subject of mathematical and scientific study. From the perspective of Computer Engineering and Computer Science, not only does it serve as an important collaborative area in such scientific research, but the conjecture algorithm is also utilized in various practical application domains. These applications of the Collatz Conjecture offer innovative solutions, particularly in fields such as data security, encryption, and digital authentication. Recent studies on the mathematical foundation and application areas of the conjecture provide new perspectives on both the mathematical resolution of the problem and its potential technological uses. In this study, some mathematical expressions derived from the Collatz Procedure are presented, and a method is proposed to determine these expressions algorithmically. The proposed algorithm has been demonstrated through examples. These expressions and the algorithm may prove useful in both the proof and practical applications of the Collatz Conjecture.

Keywords: Collatz Conjecture, Collatz Procedure, Mathematical Representation, Algorithmic Approach.
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1 Introduction

According to the conjecture proposed by L. Collatz in 1937, "Numbers entered into the Collatz function always reach 1" (Lagarias, 2023). Numerous studies have been conducted to prove the hypothesis. Important mathematicians have worked on it for a long time, and the problem has been investigated from different angles (Lagarias, 2023).

Nuriyeva & Nuriyev (2017) presents a representation of the problem in the base-4 number system, and (Nuriyeva & Nuriyev, 2024) provides a graph representation of the problem based on the results in Nuriyeva & Nuriyev (2017).

The most important recent work on the Collatz Conjecture is by Terence Tao. In his 2019 paper, he proved using probability that almost all Collatz orbits are bounded by any function that diverges to infinity (Tao, 2022).

This problem has found various applications in fields such as computer science, cryptography, and steganography, beyond being merely a mathematical hypothesis (Wirsching, 1998). These applications of the Collatz Conjecture offer innovative solutions, particularly in areas such as data security, encryption, and digital identity verification (Tuncer & Kurum, 2022).

2 Collatz Conjecture

2.1 Definition of the problem

Definition: For any positive integer n, when it is input into the function $f(n)$ defined as below, it reaches the number 1 after a finite number of iterations.

If the number n given to the function is odd, it is multiplied by 3 and 1 is added; if the number n is even, it is divided by 2. The iteration continues until the number 1 is reached. The function described in this definition is expressed as follows:

$$f : \mathbb{Z}^+ \to \mathbb{Z}^+, \quad f_0 = n, \quad n \neq 0$$

$$f_{i+1} = \begin{cases} \dfrac{f_i}{2}, & \text{if } f_i \text{ is even,} \\ 3f_i + 1, & \text{if } f_i \text{ is odd.} \end{cases}$$

The Collatz Conjecture predicts that a positive integer n will reach 1 through iterations of a certain function.

2.2 Collatz function

In mathematical notation, the Collatz function can be expressed as follows:

$$C : \mathbb{Z}^+ \to \mathbb{Z}^+$$

$$C(n) = \begin{cases} \dfrac{n}{2}, & \text{if } n \equiv 0 \pmod 2, \\ 3n + 1, & \text{if } n \equiv 1 \pmod 2. \end{cases}$$

$$F(n) = \{\, n,\, C(n),\, C(C(n)),\, C(C(C(n))), \dots \} = \{\, n,\, C(n),\, C^2(n),\, C^3(n), \dots \}.$$

The function starts with the initial value n and always reaches 1 after iterations. This procedure forms the basis of the Collatz Conjecture.

2.3 Collatz Procedure

The step-by-step process of the Collatz function is as follows:

1. A positive integer is selected.
2. If the number is even, it is divided by two.
3. If the number is odd, it is first multiplied by three, one is added, and then it is divided by two.
4. This process continues until the result is 1.

For example, let's take the number 3 (CP(3)):

1. 3 is odd $\to$ $3(3) + 1 = 10$
2. 10 is even $\to$ $10/2 = 5$
3. 5 is odd $\to$ $3(5) + 1 = 16$
4. 16 is even $\to$ $16/2 = 8$

5. 8 is even $\rightarrow$ $8/2 = 4$
6. 4 is even $\rightarrow$ $4/2 = 2$
7. 2 is even $\rightarrow$ $2/2 = 1$

Result: The number 3 reaches 1 in 7 steps.

Now let's examine the series calculated for numbers from 1 to 10;

$C(1) = \{1\}$,

$C(2) = \{2, 1\}$,

$C(3) = \{3, 10, 5, 16, 8, 4, 2, 1\}$,

$C(4) = \{4, 2, 1\}$,

$C(5) = \{5, 16, 8, 4, 2, 1\}$,

$C(6) = \{6, 3, 10, 5, 16, 8, 4, 2, 1\}$,

$C(7) = \{7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1\}$,

$C(8) = \{8, 4, 2, 1\}$,

$C(9) = \{9, 28, 14, 7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1\}$,

$C(10) = \{10, 5, 16, 8, 4, 2, 1\}$.

2.4 Conjecture notation

For every $n \in \mathbb{Z}^+$ there exists a $k \in \mathbb{Z}^+$ such that $C^k(n) = 1$

$\forall n \in \mathbb{Z}^+,\ \exists k \in \mathbb{Z}^+$ such that $C^k(n) = 1$

3 Some Mathematical Representations for the Collatz Conjecture

In the following sections of our article, we will ignore even numbers, and all operations will be performed only with odd numbers. For example, if we consider the Collatz Procedure for the number $n = 9$, we generally reach the number 1 in 19 steps:

$C(9) = \{9, 28, 14, 7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1\}$

But if we only show this procedure with odd numbers, we will reach the number 1 in 6 steps:

$CO(9) = \{9, 7, 11, 17, 13, 5, 1\}$

Here, the part of the CO and C sets with odd numbers is shown. Now, let's look at the

CO series calculated for odd numbers from 1 to 10:

$CO(1) = \{1\}$,

$CO(3) = \{3, 5, 1\}$,

$CO(5) = \{5, 1\}$,

$CO(7) = \{7, 11, 17, 13, 5, 1\}$,

$CO(9) = \{9, 7, 11, 17, 13, 5, 1\}$.

(Nuriyeva & Nuriyev, 2025) demonstrated that the following theorem is true:

Theorem 1. *For every number n, the following statement is true:*

$$3^m \cdot n + 3^{m-1} + 3^{m-2} \cdot 2^{k_{m-2}} + 3^{m-3} \cdot 2^{k_{m-3}} + \cdots + 3^2 \cdot 2^{k_2} + 3^1 \cdot 2^{k_1} + 3^0 \cdot 2^{k_0} = 2^k \qquad (1)$$

Here, m is the number of steps in $CO(n)$, $k_0 = k - 4$, and $k_0 > k_1 > k_2 > \cdots > k_{m-2}$.

Below is the expansion of expression (1) for all odd numbers in the range $n = 1$–27:

$CPMR(\mathbf{1}) = 3^1 \cdot \mathbf{1} + 3^0 = 2^2$

$CPMR(\mathbf{3}) = 3^2 \cdot \mathbf{3} + 3^1 + 3^0 \cdot 2^1 = 2^5$

$CPMR(\mathbf{5}) = 3^1 \cdot \mathbf{5} + 3^0 = 2^4$

$CPMR(\mathbf{7}) = 3^5 \cdot \mathbf{7} + 3^4 + 3^3 \cdot 2 + 3^2 \cdot 2^2 + 3^1 \cdot 2^4 + 2^7 = 2^{11}$

$CPMR(\mathbf{9}) = 3^6 \cdot \mathbf{9} + 3^5 + 3^4 \cdot 2^2 + 3^3 \cdot 2^3 + 3^2 \cdot 2^4 + 3 \cdot 2^6 + 2^9 = 2^{13}$

$CPMR(\mathbf{11}) = 3^4 \cdot \mathbf{11} + 3^3 + 3^2 \cdot 2^1 + 3 \cdot 2^3 + 2^6 = 2^{10}$

$CPMR(\mathbf{13}) = 3^2 \cdot \mathbf{13} + 3 + 2^3 = 2^7$

$CPMR(\mathbf{15}) = 3^5 \cdot \mathbf{15} + 3^4 + 3^3 \cdot 2 + 3^2 \cdot 2^2 + 3 \cdot 2^3 + 2^8 = 2^{12}$

$CPMR(\mathbf{17}) = 3^3 \cdot \mathbf{17} + 3^2 + 3 \cdot 2^2 + 2^5 = 2^9$

$CPMR(\mathbf{19}) = 3^6 \cdot \mathbf{19} + 3^5 + 3^4 \cdot 2^1 + 3^3 \cdot 2^4 + 3^2 \cdot 2^5 + 3 \cdot 2^7 + 2^{10} = 2^{14}$

$CPMR(\mathbf{21}) = 3^1 \cdot \mathbf{21} + 3^0 = 2^6$

$CPMR(\mathbf{23}) = 3^4 \cdot \mathbf{23} + 3^3 + 3^2 \cdot 2^1 + 3 \cdot 2^2 + 2^7 = 2^{11}$

$CPMR(\mathbf{25}) = 3^7 \cdot \mathbf{25} + 3^6 + 3^5 \cdot 2^2 + 3^4 \cdot 2^3 + 3^3 \cdot 2^6 + 3^2 \cdot 2^7 + 3 \cdot 2^9 + 2^{12} = 2^{16}$

$$
\begin{aligned}
CPMR(\mathbf{27}) = {} & 3^{41} \cdot \mathbf{27} + 3^{40} + 3^{39} \cdot 2 + 3^{38} \cdot 2^{3} + 3^{37} \cdot 2^{4} + 3^{36} \cdot 2^{5} + 3^{35} \cdot 2^{6} \\
& + 3^{34} \cdot 2^{7} + 3^{33} \cdot 2^{9} + 3^{32} \cdot 2^{11} + 3^{31} \cdot 2^{12} + 3^{30} \cdot 2^{14} + 3^{29} \cdot 2^{15} + 3^{28} \cdot 2^{16} \\
& + 3^{27} \cdot 2^{18} + 3^{26} \cdot 2^{19} + 3^{25} \cdot 2^{20} + 3^{24} \cdot 2^{21} + 3^{23} \cdot 2^{23} + 3^{22} \cdot 2^{26} + 3^{21} \cdot 2^{27} \\
& + 3^{20} \cdot 2^{28} + 3^{19} \cdot 2^{30} + 3^{18} \cdot 2^{31} + 3^{17} \cdot 2^{33} + 3^{16} \cdot 2^{34} + 3^{15} \cdot 2^{35} + 3^{14} \cdot 2^{36} \\
& + 3^{13} \cdot 2^{37} + 3^{12} \cdot 2^{38} + 3^{11} \cdot 2^{41} + 3^{10} \cdot 2^{42} + 3^{9} \cdot 2^{43} + 3^{8} \cdot 2^{44} + 3^{7} \cdot 2^{48} \\
& + 3^{6} \cdot 2^{50} + 3^{5} \cdot 2^{52} + 3^{4} \cdot 2^{56} + 3^{3} \cdot 2^{59} + 3^{2} \cdot 2^{60} + 3 \cdot 2^{61} + 2^{66} \\
= {} & 2^{70}
\end{aligned}
$$

4 Algorithmic Approach for Mathematical Representation

Below, a method for finding the above mathematical expressions provided by the Collatz Procedure using an algorithmic approach will be discussed.

First, let us consider the numbers in the CO set that reach 1 in one step (i.e., $m = 1$) among the odd numbers up to 27 (these are $n = 1, 5$, and 21):

$$n = \mathbf{1} \to 3 \cdot \mathbf{1} + 1 = 4 \to 3^1 \cdot \mathbf{1} + 3^0 = 2^2$$

$$n = \mathbf{5} \to 3 \cdot \mathbf{5} + 1 = 15 + 1 = 16 \to 3^1 \cdot \mathbf{5} + 3^0 = 2^4$$

$$n = \mathbf{21} \to 3 \cdot \mathbf{21} + 1 = 63 + 1 = 64 \to 3^1 \cdot \mathbf{21} + 3^0 = 2^6$$

Since $m = 1$ for the numbers 1, 5, and 21 above, the formulas can be found directly and no algorithm is required.

Let us consider the numbers that reach 1 in two steps (i.e., $m = 2$) within the CO set among the odd numbers up to 27 (these are $n = 3$ and 13):

The operation of the Collatz Procedure for $n = 3$ is given above in $CP(3)$, and its mathematical representation is given in $CPMR(3)$. Now, let us try to construct the $CPMR(3)$ formula algorithmically, starting from the value $m = 1$, step by step:

$$3 \cdot \mathbf{3} + 1 = 10 = 8 + 2 = 2^3 + 2$$

$$3 \cdot \mathbf{3} + 1 - 2 = 2^3$$

If we multiply the left side of the last equation by 3 and add the right side, we get the following:

$$3\,(3 \cdot \mathbf{3} + 1 - 2) + 2^3 = 3 \cdot 2^3 + 2^3 = 4 \cdot 2^3 = 2^2 \cdot 2^3 = 2^5$$

$$3^2 \cdot \mathbf{3} + 3 - 2 \cdot 3 + 2^3 = 2^5$$

$$3^2 \cdot \mathbf{3} + 3 + (-6 + 8) = 2^5$$

$$3^2 \cdot \mathbf{3} + 3 + 2 = 2^5$$

$$3^2 \cdot \mathbf{3} + 3^1 + 3^0 \cdot 2^1 = 2^5$$

Thus, we obtained the $CPMR(3)$ formula for $m = 2$.

The mathematical representation of the Collatz Procedure for $n = 13$ is given in $CPMR(13)$. Now, let's try to construct the $CPMR(13)$ formula algorithmically, starting from $m = 1$, step by step:

$$3 \cdot \mathbf{13} + 1 = 40 = 32 + 8 = 2^5 + 8$$

$$3 \cdot \mathbf{13} + 1 - 8 = 2^5$$

If we multiply the left side of the last equation by 3 and add the right side, we get the following:

$$3 \cdot (3 \cdot \mathbf{13} + 1 - 8) + 2^5 = 3 \cdot 2^5 + 2^5 = 4 \cdot 2^5 = 2^2 \cdot 2^5 = 2^7$$

$$3^2 \cdot \mathbf{13} + 3 - 3 \cdot 8 + 2^5 = 2^7$$

$$3^2 \cdot \mathbf{13} + 3 + (-24 + 32) = 2^7$$

$$3^2 \cdot \mathbf{13} + 3 + 8 = 2^7$$

$$3^2 \cdot \mathbf{13} + 3^1 + 3^0 \cdot 2^3 = 2^7$$

Thus, we obtained the $CPMR(13)$ formula for $m = 2$.

Only the number 17 is the number that reaches 1 in 3 steps (i.e., $m = 3$) within the CO set of odd numbers up to 27. Let's consider the number $n = 17$:

The mathematical representation of the Collatz Procedure for $n = 17$ is given in $CPMR(17)$. Now, let us try to construct the $CPMR(17)$ formula algorithmically, starting from $m = 1$, step by step:

$$3 \cdot \mathbf{17} + 1 = 52 = 32 + 20 = 2^5 + 20$$

$$3 \cdot \mathbf{17} + 1 - 20 = 2^5$$

If we multiply the left side of the last equation by 3 and add the right side, we get the following:

$$3 \cdot (3 \cdot \mathbf{17} + 1 - 20) + 2^5 = 3 \cdot 2^5 + 2^5 = 4 \cdot 2^5 = 2^2 \cdot 2^5 = 2^7$$

$$3^2 \cdot \mathbf{17} + 3 - 3 \cdot 20 + 2^5 = 2^7$$

$$3^2 \cdot \mathbf{17} + 3 + (-60 + 32) = 2^7$$

$$3^2 \cdot \mathbf{17} + 3 - 28 = 2^7$$

Since the third term in the above equation is a negative number (-28), the algorithm continues its work. That is, if we multiply the left side of the final equation by 3 again and add the right side, we get the following:

$$3 \cdot (3^2 \cdot \mathbf{17} + 3 - 28) + 2^7 = 3 \cdot 2^7 + 2^7 = 4 \cdot 2^7 = 2^2 \cdot 2^7 = 2^9$$

$$3^3 \cdot \mathbf{17} + 3^2 - 3 \cdot 28 + 2^7 = 2^9$$

$$3^3 \cdot \mathbf{17} + 3^2 + (-84 + 128) = 2^9$$

$$3^3 \cdot \mathbf{17} + 3^2 + 44 = 2^9$$

$$3^3 \cdot \mathbf{17} + 3^2 + 3 \cdot 2^2 + 2^5 = 2^9$$

Thus, we obtained the $CPMR(17)$ formula for $m = 3$.

Let us consider the numbers that reach 1 in 4 steps (i.e., $m = 4$) within the CO set among the odd numbers up to 27 (these are $n = 11$ and 23):

The mathematical representation of the Collatz Procedure for $n = 11$ is given in $CPMR(11)$. Now, let's try to construct the $CPMR(11)$ formula algorithmically, starting from $m = 1$, step by step:

$$3 \cdot \mathbf{11} + 1 = 34 = 32 + 2 = 2^5 + 2$$

$$3 \cdot \mathbf{11} + 1 - 2 = 2^5$$

If we multiply the left side of the last equation by 3 and add the right side, we get the following:

$$3 \cdot (3 \cdot \mathbf{11} + 1 - 2) + 2^5 = 3 \cdot 2^5 + 2^5 = 4 \cdot 2^5 = 2^2 \cdot 2^5 = 2^7$$

$$3^2 \cdot \mathbf{11} + 3 - 3 \cdot 2 + 2^5 = 2^7$$

$$3^2 \cdot \mathbf{11} + 3 + (-6 + 32) = 2^7$$

$$3^2 \cdot \mathbf{11} + 3 + 26 = 2^7$$

In the above combination, there are 3 positive numbers (26) and since it cannot be expressed as 2 to the power of (i.e., $3^0 \cdot 2^{k_0}$), the algorithm continues by going back, meaning that we will start with the smaller number (24) in the first step:

$$3 \cdot \mathbf{11} + 1 = 34 = 16 + 18 = 2^4 + 18$$

$$3 \cdot \mathbf{11} + 1 - 18 = 2^4$$

If we multiply the left side of the last equation by 3 and add the right side, we get the following:

$$3 \cdot (3 \cdot \mathbf{11} + 1 - 18) + 2^4 = 3 \cdot 2^4 + 2^4 = 4 \cdot 2^4 = 2^2 \cdot 2^4 = 2^6$$

$$3^2 \cdot \mathbf{11} + 3 - 3 \cdot 18 + 2^4 = 2^6$$

$$3^2 \cdot \mathbf{11} + 3 + (-54 + 16) = 2^6$$

$$3^2 \cdot \mathbf{11} + 3 - 38 = 2^6$$

Since the third number in the above equation is a negative number (-38), the algorithm continues its work. That is, if we multiply the left side of the final equation by 3 again and add the right side, we get the following:

$3 \cdot (3^2 \cdot \mathbf{11} + 3 - 38) + 2^6 = 3 \cdot 2^6 + 2^6 = 4 \cdot 2^6 = 2^2 \cdot 2^6 = 2^8$

$3^3 \cdot \mathbf{11} + 3^2 - 3 \cdot 38 + 2^6 = 2^8$

$3^3 \cdot \mathbf{11} + 3^2 + (-114 + 64) = 2^8$

$3^3 \cdot \mathbf{11} + 3^2 - 50 = 2^8$

Since the third term in the above equation is a negative number (-50), the algorithm continues its work. That is, if we multiply the left side of the final equation by 3 again and add the right side, we get the following:

$3 \cdot (3^3 \cdot \mathbf{11} + 32 - 50) + 2^8 = 3 \cdot 2^8 + 2^8 = 4 \cdot 2^8 = 2^2 \cdot 2^8 = 2^{10}$

$3^4 \cdot \mathbf{11} + 3^3 - 3 \cdot 50 + 2^8 = 2^{10}$

$3^4 \cdot \mathbf{11} + 3^3 + (-150 + 256) = 2^{10}$

$3^4 \cdot \mathbf{11} + 3^3 + 106 = 2^{10}$

$3^4 \cdot \mathbf{11} + 3^3 + 3^2 \cdot 2^1 + 3 \cdot 2^3 + 2^6 = 2^{10}$

Thus, we obtained the $CPMR(11)$ formula for $m = 4$.

5 Algorithm

The following algorithm is recommended to find the mathematical expressions provided by the Collatz Procedure for any given number n:

5.1 CA Algorithm

CA1: $m = 1$
CA2: $3\boldsymbol{n} + 1 = A_m$ is calculated. If $A_m = 2^k$, go to CA9.
CA3: $m = m + 1$
CA4: Find the number k that satisfies the condition $2^k < A_m < 2^{k+1}$.
CA5: $3^m\boldsymbol{n} + 3^{m-1} = 2^k + a_m$ is shown.

From here, we can write the following (2) equality:

$$3^m\boldsymbol{n} + 3^{m-1} - a_m = 2^k \tag{2}$$

CA6: If we multiply the left side of the last equation by 3 and add the right side, we get the following:

$3\left(3^m\boldsymbol{n} + 3^{m-1} - a_m\right) + 2^k = 3 \cdot 2^k + 2^k = 4 \cdot 2^k = 22 \cdot 2^k = 2^{k+2}$

$3^{m+1} \cdot \boldsymbol{n} + 3^m - 3 \cdot a_m + 2^k = 2^{k+2}$

$3^{m+1} \cdot \boldsymbol{n} + 3^m + (-3 \cdot a_m + 2^k) = 2^{k+2}$

$D_m = -3 \cdot a_m + 2^k$

$3^{m+1} \cdot \boldsymbol{n} + 3^m + D_m = 2^{k+2}$

CA7: If $D_m < 0$, then $m = m + 1$ and go to CA6.
CA8: If

$$D_m = 3^{m-2} \cdot 2^{k_{m-2}} + 3^{m-3} \cdot 2^{k_{m-3}} + \cdots + 3^2 \cdot 2^{k_2} + 3^1 \cdot 2^{k_1} + 3^0 \cdot 2^{k_0},$$

set $k = k - 1$ and proceed to CA5.
CA9: The number $\boldsymbol{n}$ reaches 1 after m iterations and can be written as formula (1) according to the found m and k.

5.2 Example: How the algorithm works for $n = 23$

Now, let's apply the above algorithm for $n = 23$ to create the formula $CPMR(23)$:

$3 \cdot \mathbf{23} + 1 = 70$

$\forall k \in \mathbb{Z}, \quad 70 \neq 2^k \;\Rightarrow\; m \neq 1, \quad m = m + 1, \quad m = 2\ .$

From the condition $2^6 < 70 < 2^7$, $k = 6$ is found, and the two terms are written as follows:

$3 \cdot \mathbf{23} + 1 = 70 = 64 + 6 = 2^6 + 6$

$$3 \cdot \mathbf{23} + 1 - 6 = 2^6 \tag{2'}$$

From here, $D_2 = -6$.

If we multiply the left side of the equation (2′) by 3 and add the right side, we get the following:

$3 \cdot (3 \cdot \mathbf{23} + 1 - 4) + 2^6 = 3 \cdot 2^6 + 2^6 = 4 \cdot 2^6 = 2^2 \cdot 2^6 = 2^8$

$3^2 \cdot \mathbf{23} + 3 - 3 \cdot 4 + 2^6 = 2^8$

$3^2 \cdot \mathbf{23} + 3 + (-18 + 64) = 2^8$

$3^2 \cdot \mathbf{23} + 3 + 46 = 2^8$

In the above combination, there are 3 positive numbers (46) and since it cannot be expressed as 2 to the power of (i.e., $3^0 \cdot 2^{k_0}$), the algorithm continues by going back, meaning that we will start with the smaller number ($2^6 - 1 = 2^5$) in the first step:

$3 \cdot \mathbf{23} + 1 = 70 = 32 + 38 = 2^5 + 38$

$3 \cdot \mathbf{23} + 1 - 38 = 2^5$

If we multiply the left side of the last equation by 3 and add the right side, we get the following:

$3 \cdot (3 \cdot \mathbf{23} + 1 - 38) + 2^5 = 3 \cdot 2^5 + 2^5 = 4 \cdot 2^5 = 2^2 \cdot 2^5 = 2^7$

$3^2 \cdot \mathbf{23} + 3 - 3 \cdot 38 + 2^5 = 2^7$

$3^2 \cdot \mathbf{23} + 3 + (-114 + 32) = 2^7$

$3^2 \cdot \mathbf{23} + 3 - 82 = 2^7$

Since the third term in the above equation is a negative number (-82), the algorithm continues its work. That is, if we multiply the left side of the final equation by 3 again and add the right side, we get the following:

$3 \cdot (3^2 \cdot \mathbf{23} + 3 - 82) + 2^7 = 3 \cdot 2^7 + 2^7 = 4 \cdot 2^7 = 2^2 \cdot 2^7 = 2^9$

$3^3 \cdot \mathbf{23} + 3^2 - 3 \cdot 82 + 2^7 = 2^9$

$3^3 \cdot \mathbf{23} + 3^2 + (-246 + 128) = 2^9$

$3^3 \cdot \mathbf{23} + 3^2 - 118 = 2^9$

Since the third term in the above equation is a negative number (-118), the algorithm continues its work. That is, if we multiply the left side of the final equation by 3 again and add the right side, we get the following:

$3 \cdot (3^3 \cdot \mathbf{23} + 3^2 - 118) + 2^9 = 3 \cdot 2^9 + 2^9 = 4 \cdot 2^9 = 2^2 \cdot 2^9 = 2^{11}$

$3^4 \cdot \mathbf{23} + 3^3 - 3 \cdot 118 + 2^9 = 2^{11}$

$3^4 \cdot \mathbf{23} + 3^3 + (-354 + 512) = 2^{11}$

$3^4 \cdot \mathbf{23} + 3^3 + 158 = 2^{11}$

$3^4 \cdot \mathbf{23} + 3^3 + 3^2 \cdot 2^1 + 3 \cdot 2^2 + 2^7 = 2^{11}$

Thus, we obtained the $CPMR(23)$ formula for $m = 4$.

6 Conclusion

The Collatz Conjecture has not yet been completely solved mathematically. The problem is being researched from different angles. From this perspective, we believe that the proposed algorithm will be useful.

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