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COMPARATIVE ANALYSIS OF THE MAJOR TOPSIS-RELATED ALGORITHMS

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Abstract. The Technique of Ordering Preference by Similarity to Ideal Solution (TOPSIS) method is one of the basic decision-making (DM) methods used in multi-criteria decision-making (MCDM) problems. In this study, various variants of the TOPSIS method are comparatively analyzed from algorithmic perspectives. The TOPSIS-based methods adapted to various application areas in the last years, especially in the last decade are discussed. As can be seen from literature studies, TOPSIS approaches with fuzzy information are widely used. Triangular, trapezoidal and sometimes Gaussian fuzzy numbers are used in processing fuzzy information. Details of various TOPSIS approaches are discussed and comparative analysis is made from these and similar perspectives.

Keywords: Multi-criteria decision-making (MCDM), TOPSIS, Fuzzy TOPSIS.

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1 Introduction

The Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) is a multi-criteria decision-making (MCDM) method, which is one of the most frequently used methods in solving MCDM problems. TOPSIS is based on the concept that the chosen alternative should have the shortest distance from the positive ideal solution (PIS) and the longest geometric distance from the negative ideal solution (NIS). The TOPSIS method was originally developed in Hwang & Yoon (1981) with further developments in Yoon (1987), and Hwang et al. (1993). Fuzzy TOPSIS was proposed in Chen (2000). Later fuzzy similarity-based approach to TOPSIS and various extensions to forming ideal solutions was proposed in the literature.

Although nearly 40 years have passed since it was first proposed, the TOPSIS method is still frequently applied and developed by researchers. Surveys of studies related to the TOPSIS method are surveyed at various intervals. There are comparisons between different modifications of TOPSIS in some studies (Madi et al., 2015). However, these survey studies generally taxonomize the TOPSIS method in terms of application areas (Salih et al., 2019; Nadaban et al., 2016; Behzadian et al., 2012; Pereira & Bamel, 2012).

In our study, the AHP and TOPSIS-related methods, especially the fuzzy TOPSIS method, is discussed from an algorithmic point of view and the studies in the literature are selected and examined in this respect. In the study, the literature was searched using Google Scholar using "fuzzy TOPSIS, fuzzy VICOR, fuzzy AHP" keys. Studies covering the last 10 years were mainly examined. Among the found articles, studies with certain algorithmic differences were selected and analyzed in more detail.

Rest of the paper is organized as follows. Algorithmical foundations of the TOPSIS method is given in section 2. Section 3 describes variants of the TOPSIS with different similarity-based and fuzzy approaches. A comparative analysis of the TOPSIS-based approaches is given in Section 4. Finally, the Conclusion section finalizes the paper.

2 Algorithmical Foundation of the TOPSIS

Various combinations of fuzzy AHP (Analytical Hierarchy Process), TOPSIS, BWM (Best-Worst Method) and VIKOR (VIseKriterijumska Optimizacija I Kompromisno Resenje) methods are discussed in the reviewed articles. Algorithmically, all articles are almost the same. Triangular or trapezoidal fuzzy numbers are generally used in fuzzy TOPSIS algorithms, and criterion weights are determined primarily by AHP or BWM methods. Then TOPSIS or VIKOR are used to rank the alternatives. In the AHP method, row normalization or optimization (entropy based, logarithmic etc.) approaches are used to determine the weights of the criteria. Different fuzzy distances are used between the best and worst fuzzy values in the methods used. In the fuzzy TOPSIS method, the closeness coefficient is calculated using fuzzy arithmetical operations.

The detailed TOPSIS process can be presented as follows:

Step 1. Create an evaluation matrix consisting of m alternatives and n criteria, with the intersection of each alternative and criteria given as x_{ij}, we therefore have a matrix $(x_{ij})_{m\times n}$.

Step 2. The matrix $(x_{ij})_{m\times n}$ is then normalised to form the matrix $R = (r_{ij})_{m\times n}$ using the normalisation method

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{k=1}^{m} x_{kj}^2}}, \quad i = 1, \ldots, m; j = 1, \ldots, n.$$

Step 3. Calculate the weighted normalised decision matrix

$$t_{ij} = r_{ij} \cdot w_j, \quad i = 1, \ldots, m; j = 1, \ldots, n,$$

where

$$w_j = \frac{W_j}{\sum_{k=1}^{n} W_k}, \quad j = 1, \ldots, n,$$

so that $\sum_{k=1}^{n} w_k = 1$ and W_k is the original weight given to the criterion k.

Step 4. Determine the worst alternative A_w (the negative ideal solution (NIS)) and the best alternative A_b (the positive ideal solution (PIS)):

$$A_w = \{(\max(t_{ij}|i = 1, \ldots, m; j \in J_-), (\min(t_{ij}|i = 1, \ldots, m; j \in J_+))\} \equiv \{t_{wj}|j = 1, \ldots, n\},$$
$$A_b = \{(\min(t_{ij}|i = 1, \ldots, m; j \in J_-), (\max(t_{ij}|i = 1, \ldots, m; j \in J_+))\} \equiv \{t_{bj}|j = 1, \ldots, n\},$$

where,

$$J_+ = \{j = 1, \ldots, n \mid j\text{th criteria is a benefit criterion}\},$$
$$J_- = \{j = 1, \ldots, n \mid j\text{th criteria is a cost criterion}\}.$$

Step 5. Calculate the L^2-distance between the alternative i and the worst condition A_w, i.e. the distance from the negative ideal solution:

$$d_{iw} = \sqrt{\sum_{j=1}^{n} (t_{ij} - t_{wj})^2}, \quad i = 1, \ldots, m,$$

and the distance between the alternative i and the best condition A_b, i.e. the distance from the positive ideal solution:

$$d_{ib} = \sqrt{\sum_{j=1}^{n} (t_{ij} - t_{bj})^2}, \quad i = 1, \ldots, m,$$

where d_{iw} and d_{ib} are L^2-norm distances from the target alternative i to the worst and best conditions, respectively.

Step 6. Calculate the normalized similarity to the best condition:

$$s_{ib} = 1 - \frac{d_{ib}}{d_{iw} + d_{ib}}, \quad 0 \le s_{iw} \le 1, \quad i = 1, \ldots, m.$$

So, $s_{ib} = 1$, if and only if the alternative solution has the best condition; and $s_{ib} = 0$, if and only if the alternative solution has the worst condition.

Step 7. Rank the alternatives according to s_{ib} $(i = 1, \ldots, m)$.

Thus, the steps of the basic TOPSIS algorithm can be summarized from a broader perspective as follows:

Step 1. Determination of sets of criteria and alternatives.
Step 2. Determination of criteria weights.
Step 3. Creating a decision matrix containing the evaluation scores of each alternative according to each criterion.
Step 4. Normalizing the decision matrix.
Step 5. Creating a weighted normalized matrix, taking into account the weights of the criteria.
Step 6. Creation of PIS and NIS vectors.
Step 7. Calculating the distances of each alternative from the PIS and NIS vectors.
Step 8. Calculating the Closeness Coefficient (CC) values for each alternative according to the PIS and NIS distances.
Step 9. Ranking the alternatives according to their CC values. Alternatives with a higher CC value are preferable.

3 Variants of the TOPSIS method

3.1 General steps

Different aspects of the articles using the TOPSIS algorithm are determined depending on the different execution of the steps in the basic TOPSIS method.

Step 1. Step 1 is usually carried out depending on the application area. In the articles discussed in this survey, the Fuzzy TOPSIS method has been used in various application areas are discussed such as risk assessment, information source selection, emergency evaluation, electronic service quality in healthcare industry, organizational strategy development, projects selection, etc.

Step 2. In order to perform Step 2, methods such as AHP, BWM before TOPSIS are generally used. In the fuzzy TOPSIS algorithm, the weight values can be in the form of fuzzy numbers. The weights are determined by Fuzzy AHP, Fuzzy BWM etc. methods. They can also be given directly by the expert. Generally, Triangular and Trapezoidal Fuzzy Numbers (FN) are used for this purpose. Fuzzy division operation is performed in generating the transpose values according to the diagonal of the matrix. In the case of MCDM, weights are created for each decision maker (DM) and then converted into a single vector of aggregated weights. For this purpose, average or different aggregation functions can be used. For example, in the case of trapezoidal fuzzy numbers in the form of (a_i, b_i, c_i, d_i), the aggregated values of the multi-criteria evaluation results $(\overline{a_i}, \overline{b_i}, \overline{c_i}, \overline{d_i})$ can be calculated as follows:

$$\overline{a_i} = \min_k a_k, \overline{b_i} = \frac{1}{K}\sum_{k=1}^{K} b_k, \overline{c_i} = \frac{1}{K}\sum_{k=1}^{K} c_k, \overline{d_i} = \max_k d_k. \tag{1}$$

Step 3. Fuzzy TOPSIS algorithm differs from the classical TOPSIS algorithm according to the value types contained in the matrix containing alternative evaluation scores (decision matrix) according to the criteria in step 3. In the fuzzy TOPSIS algorithm, these values are in the form of fuzzy numbers. Usually Triangular FN and Trapezoidal FN numbers are used. In the case of MCDM, this matrix is created for each DM and then converted into a single aggregated matrix. For this purpose, the arithmetic mean or different aggregation functions can be used. For example, the approach in eq. (1) can be used.

Step 4. In step 4, normalization is used in the classical TOPSIS algorithm as follows:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{k=1}^{m} x_{kj}^2}}, \quad i = 1, \ldots, m; j = 1, \ldots, n.$$

But in the case of the Fuzzy TOPSIS, the normalized fuzzy decision matrix can be represented as $\widehat{R} = (\widehat{r}_{ij})_{m \times n}$ where B and C are the sets of benefit criteria and cost criteria, respectively, and

$$\widehat{r}_{ij} = \left(\frac{a_{ij}}{d_j^{\oplus}}, \frac{b_{ij}}{d_j^{\oplus}}, \frac{c_{ij}}{d_j^{\oplus}}, \frac{d_{ij}}{d_j^{\oplus}} \right), \quad j \in B,$$

for benefit criteria and

$$\widehat{r}_{ij} = \left(\frac{a_j^{\ominus}}{d_{ij}}, \frac{a_j^{\ominus}}{c_{ij}}, \frac{a_j^{\ominus}}{b_{ij}}, \frac{a_j^{\ominus}}{a_{ij}} \right), \quad j \in C,$$

for cost criteria, where

$$d_j^{\oplus} = \max_i d_{ij}, \quad j \in B,$$
$$a_j^{\ominus} = \min_i a_{ij}, \quad j \in C.$$

Step 5. The calculation of step 5 is performed by multiplying the criteria weights with the values of the decision matrix that comply with this criterion. In the case of fuzzy TOPSIS, multiplication between fuzzy numbers is used.

Step 6. In the classical TOPSIS algorithm, the vector consisting of the max (for the benefit criterion) and min (for the cost criterion) values of each column of the weighted normalized decision matrix is used for the PIS vector according to Step 6. For the NIS, the vector of min (for benefit criteria) and max (for cost criteria) values of each column of the normalized decision matrix is used. We will refer to this as the max-min approach in our survey. Most studies use the vector $(1, 1, \ldots, 1)$ for PIS and $(0, 0, \ldots, 0)$ for NIS instead. We will refer to this approach as the 0-1 approach in our article.

In the case where the weighted normalized decision metric consists of fuzzy numbers such as $v_{ij} = (v_{ij1}, v_{ij2}, v_{ij3}, v_{ij4})$, the Fuzzy PIS (FPIS) and Fuzzy NIS (FNIS) vectors can be created as follows:

$$v_j^+ = \left(v_{j1}^+, v_{j2}^+, v_{j3}^+, v_{j4}^+\right) = \left(\max_i v_{ij1}, \max_i v_{ij2}, \max_i v_{ij3}, \max_i v_{ij4} \right),$$
$$v_j^- = \left(v_{j1}^-, v_{j2}^-, v_{j3}^-, v_{j4}^-\right) = \left(\min_i v_{ij1}, \min_i v_{ij2}, \min_i v_{ij3}, \min_i v_{ij4} \right).$$

Step 7. In the classical TOPSIS algorithm, the distances of each alternative from the PIS and NIS vectors are usually calculated using the Euclidean distance. In the fuzzy TOPSIS algorithm, various fuzzy distances are used for this purpose. For example, the distance of the alternative $v_{ij} = (v_{ij1}, v_{ij2}, v_{ij3}, v_{ij4})$ to the FPIS and FNIS according to the FN parameters (the vertex distance) can be calculated as follows, respectively:

$$\mathrm{dist}(v_{ij}, v_j^+) = \sqrt{\frac{1}{4}\left[(v_{ij1} - v_{j1}^+)^2 + (v_{ij2} - v_{j2}^+)^2 + (v_{ij3} - v_{j3}^+)^2 + (v_{ij4} - v_{j4}^+)^2\right]},$$
$$\mathrm{dist}(v_{ij}, v_j^-) = \sqrt{\frac{1}{4}\left[(v_{ij1} - v_{j1}^-)^2 + (v_{ij2} - v_{j2}^-)^2 + (v_{ij3} - v_{j3}^-)^2 + (v_{ij4} - v_{j4}^-)^2\right]}.$$

Step 8. Then the closeness coefficients of the ith alternative with respect to the positive ideal solution by using the similarity measures matrix (CCS_i^+) are defined as:

$$CCS_i^+ = \frac{s_i^+}{s_i^+ + s_i^-}, \quad i = 1, \dots, m,$$

where,

$$\begin{aligned} s_i^+ &= s(v_{ij}, v_j^+) = 1 - \mathrm{dist}(v_{ij}, v_j^+), \\ s_i^- &= s(v_{ij}, v_j^-) = 1 - \mathrm{dist}(v_{ij}, v_j^-). \end{aligned}$$

Step 9. Finally, the alternatives can be ranked by their CCS_i^+ values. The alternative with a larger closeness coefficient is the preferable alternative.

3.2 Similarity based TOPSIS

In the TOPSIS method, the Euclidean distance is generally used to calculate the distances to the ideal solutions, but different distances can also be used. Also, different approaches can be used for the normalization of the R matrix in Step 2. Similarity based TOPSIS method has been proposed after the original TOPSIS method. In the similarity based TOPSIS method, the r_{ij} values in Step 2 of the basic TOPSIS algorithm are calculated as follows:

$$r_{ij} = \frac{x_{ij} - \min_i x_{ij}}{\max_i x_{ij} - \min_i x_{ij}}, \quad i = 1, \dots, m; j = 1, \dots, n.$$

The similarity values in Step 6 can be calculated as follows:

$$\begin{aligned} s_{iw} &= 1 - d_{iw}, \\ s_{ib} &= 1 - d_{ib}, \quad i = 1, \dots, n. \end{aligned}$$

The ranking of alternatives with respect to the Step 7 is made according to the closeness coefficient to positive ideal solution values calculated as follows:

$$CC_i = \frac{s_i^+}{s_i^+ + s_i^-}, \quad i = 1, \dots, n.$$

3.3 Fuzzy TOPSIS

The fuzzy similarity-based approach to Fuzzy TOPSIS and extensions to forming ideal solutions was proposed in different studies. In the fuzzy similarity TOPSIS method, x_{ij}, $i = 1, \dots, m; j = 1, \dots, n$, values are determined as fuzzy numbers. Various fuzzy numbers are used for this purpose. The most commonly used fuzzy number types are triangular and trapezoidal fuzzy numbers. In the case of trapezoidal fuzzy numbers in the form of (a_i, b_i, c_i, d_i), the aggregated values of the multi-criteria evaluation results $(\overline{a_i}, \overline{b_i}, \overline{c_i}, \overline{d_i})$ can be calculated as follows:

$$\overline{a_i} = \min_k a_k, \overline{b_i} = \frac{1}{K}\sum_{k=1}^{K} b_k, \overline{c_i} = \frac{1}{K}\sum_{k=1}^{K} c_k, \overline{d_i} = \max_k d_k.$$

However, different arithmetic operations can be used on fuzzy numbers for this purpose. Therefore, the normalized fuzzy decision matrix can be represented as $\widehat{R} = (\widehat{r}_{ij})_{m\times n}$ where B and C are the sets of benefit criteria and cost criteria, respectively, and

$$\widehat{r}_{ij} = \left(\frac{a_{ij}}{d_j^{\oplus}}, \frac{b_{ij}}{d_j^{\oplus}}, \frac{c_{ij}}{d_j^{\oplus}}, \frac{d_{ij}}{d_j^{\oplus}} \right), \quad j \in B,$$

for benefit criteria and

$$\widehat{r}_{ij} = \left(\frac{a_j^{\ominus}}{d_{ij}}, \frac{a_j^{\ominus}}{c_{ij}}, \frac{a_j^{\ominus}}{b_{ij}}, \frac{a_j^{\ominus}}{a_{ij}} \right), \quad j \in C,$$

for cost criteria, where

$$d_j^{\oplus} = \max_i d_{ij}, \quad j \in B, a_j^{\ominus} = \min_i a_{ij}, \quad j \in C.$$

Considering the importance of each criterion, the weighted normalized fuzzy decision matrix can now be expressed as $\widehat{V} = (\widehat{v}_{ij})_{m\times n}$, where

$$\widehat{v}_{ij} = \widehat{r}_{ij} \odot \widehat{w}_j.$$

where the operation $\odot$ is a fuzzy product. The obtained weighted normalized values v_{ij} are fuzzy numbers expressed as $v_{ij} = (v_{ij1}, v_{ij2}, v_{ij3}, v_{ij4})$.

There are several different approaches to determine the FPIS and FNIS values.

1. First proposal for choosing the fuzzy positive ideal solution (FPIS) (v^+) and the fuzzy negative ideal solution (FNIS) (v^-) have been given as:

$$v_j^+ = \max_i v_{ij4}, and v_j^- = \min_i v_{ij1}.$$

2. v_j^+ as a vector of ones, and v_j^- as a vector of zeros:

$$v_j^+ = (1, 1, \ldots, 1), and v_j^- = (0, 0, \ldots, 0).$$

3. Every element of v^+ is the maximum for all i weighted normalized value, and every element of v^- is the minimum for all i weighted normalized value:

$$v_j^+ = \left(\max_i v_{ij1}, \max_i v_{ij2}, \max_i v_{ij3}, \max_i v_{ij4} \right),$$
$$v_j^- = \left(\min_i v_{ij1}, \min_i v_{ij2}, \min_i v_{ij3}, \min_i v_{ij4} \right).$$

After ideal vectors are computed, next step is to calculate similarities of the ith alternative and both ideal solutions. Various distances between fuzzy numbers can be used for this purpose:

$$s_i^+ = s(v_{ij}, v_j^+) = 1 - \text{dist}(v_{ij}, v_j^+),$$
$$s_i^- = s(v_{ij}, v_j^-) = 1 - \text{dist}(v_{ij}, v_j^-).$$

For example,

$$s_i^+ = s(v_{ij}, v_j^+) = \left(1 - \frac{\sum_{t=1}^4 \text{dist}(v_{ijt}, v_{jt}^+)}{4} \right),$$
$$s_i^- = s(v_{ij}, v_j^-) = \left(1 - \frac{\sum_{t=1}^4 \text{dist}(v_{ijt}, v_{jt}^-)}{4} \right),$$

can be used. Then the closeness coefficients of the ith alternative with respect to the positive ideal solution by using the similarity measures matrix (CCS_i^+) are defined as:

$$CCS_i^+ = \frac{s_i^+}{s_i^+ + s_i^-}, \quad i = 1, \ldots, m.$$

Finally, the alternatives can be ranked by their CCS_i^+ values.

4 Comparative analysis of the major TOPSIS-related approaches

The algorithmic highlights of the studies last decade are summarized in Table 1. The reviewed studies were compared in terms of applied area, specifications used in the TOPSIS method, methods for determining the weights of the criteria, PIS and NIS determination methods, use of fuzzy distance and methods for ranking the alternatives. As can be seen from the table, the TOPSIS algorithm has been applied in a wide variety of areas. In addition to the classical TOPSIS method, it is seen that fuzzy information methods are also predominantly applied. Fuzzy information is generally considered as linguistic variables, especially triangular fuzzy numbers. In some cases, trapezoidal fuzzy numbers or fuzzy numbers with Gaussian membership functions are used. AHP or fuzzy AHP methods are often used to calculate criterion weights. In some cases, criterion weights are calculated by the BWM method.

In the calculation of PIS and NIS vectors, the max-min normalization approach is generally used. In methods with fuzzy information, FPIS and FNIS are used as the fuzzy equivalents of the PIS and NIS methods accordingly, handling fuzzy min and max operations. Various fuzzy distance concepts are used in the formation of min and max operations on fuzzy numbers. The vertex method is generally used in the calculation of fuzzy distance. In some cases, it is seen that the weighted Euclidean distance is also used. More detailed other comparison results are also given in Table 1.

5 Conclusion

Although more than 40 years have passed since the TOPSIS method, which is one of the basic methods used in solving MCDM problems, was first proposed, TOPSIS-based methods still attract the attention of researchers and various variations of this method continue to be developed. In this study, various variants of the TOPSIS method, especially fuzzy applications of the studied of the last decade, were analysed from an algorithmic perspective. We believe that this study will be useful for researchers in the development of MCDM applications for fields, especially in fuzzy information MCDM applications.

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Table 1: Algorithmic specifications of related TOPSIS studies

Article	**Applied area**	**Specifications**	**Criterial weights**	**Using FNs**	**PIS and NIS**	**Fuzzy distance**	**Final ranking**
Mangeli et al., 2019	The work utilizes a hybrid approach based on support vector machine and fuzzy inference system to decrease the effect of personal's opinions in determining the factors of the severity and occurrence.	Revised fuzzy TOPSIS used for more accurate ranking of risks.	An approach using Logarithmic Fuzzy Preference Programming (optimization like BWM) is used to determine the crisp weight of dependent risk factors.	Triangular FN	PIS and NIS are (1,1,1) and (0,0,0) respectively.	Fuzzy distance is the vertex method (based on Euclidean distance).	Closeness coefficient, mean values and shape values according to Revised TOPSIS method.
Mandic et al., 2014	The study proposes a fuzzy multi-criteria model that facilitates the assessment of the financial performance of banks.	A model was constructed combining two methods of multi-criteria decision-making: Fuzzy AHP and TOPSIS. In the first phase, the priority weights of the criteria were determined using Fuzzy AHP, while in the second phase the banks are ranked through the use of the TOPSIS method.	Criteria's weights are calculated using fuzzy comparison of FN (the degree of possibility for a convex fuzzy number). The fuzzy AHP method is used to determine the final priority of weights based on triangular fuzzy numbers	Triangular FN	FPIS and FNIS vectors were determined by the max-min approach.	Euclidean distance.	Classical TOPSIS was used to rank the alternatives
Tian et al., 2013	The paper proposes a Multi-Criteria Group Decision Making (MCGDM) model to provide an effective framework for ranking competing alternatives of Information Source selection.	Besides, the Euclidean Distance (ED) is substituted by the Value of Chi-Square Test (VCST) to refine the Relative Closeness (RC), which theoretically excluded the potential bias arising from relative importance of the two types of distances, in a revised TOPSIS.	An optimization-based approach was used to determine the criterion weights. AHP and Entropy Weights (EW) are integrated to alleviate the conflicts by experts' intuitions as trapezoidal FN and provide the accurate weight vector in this model.	Trapezoidal FN was used.	FPIS and FNIS vectors were determined by the max-min approach.	The fuzzy distance similar to the mean square of the parameter differences (vertex method) is used as the distance.	Relative closeness is used. The final optimal solution is determined via voting of DM preferences using the Weighted Borda Function (WBF).

Ju and Wang, 2012	In this paper, the DS/AHP method and extended TOPSIS method are incorporated to solve group multi-criteria decision making (GMCDM) problems with incomplete information. An emergency alternative evaluation selection problem is taken as an illustrative example to demonstrate the feasibility and practicability of the proposed methods for group decision making in emergency management.	The confidence level of each DM in case of MCDM has been taken into account. The proposed method identifies the focal elements of each decision maker according to the group decision matrix.	The group weighted normalized belief interval decision matrix using Dempster's rule of combination is constructed.	Linguistic terms.	PIS and NIS vectors were determined by min-max approach.	Euclidean distance.	An Extended TOPSIS approach for group interval data to rank the alternatives.
Buyukozkan and Cifci, 2012	This study examines the e-sq concept and determine the key components of electronic service quality (e-sq) with a web service performance example of healthcare sector in Turkey by using a combined MCDM containing fuzzy AHP and fuzzy TOPSIS.	The a-cut is known to incorporate the experts or decision maker(s) confidence over his/her preferences. The degree of satisfaction for the judgment matrix is estimated by the index of optimism.	The weights of the criteria are determined by fuzzy AHP.	Triangular FN is used.	The 0-1 approach is used for FPIS and FNIS	Fuzzy distance using vertex method.	Fuzzy TOPSIS is used to final ranking of the alternatives (Closeness coefficient).
Paksoy et al., 2012	In this paper, the methods of Fuzzy AHP and hierarchical fuzzy TOPSIS (HFTOPSIS) for evaluating and selecting among the five organization strategy models for distribution channel management of vegetable oil manufacturer have been presented.	While applying fuzzy AHP, the approach proposed by Chang is used (Chang, 1996).	The criterion weights are determined by fuzzy AHP.	Triangular fuzzy numbers are used.	The max-min approach is used to determine FPIS and FNIS.	Using the separation measures.	Alternatives are ranked via relative closeness.

Akkaya et al., 2015	In the study, a model is created for students to decide in which sector they should be work in the future. Fuzzy AHP and fuzzy MOORA (Multi-Objective Optimization by Ratio Analysis) methods have been applied to the results of the questionnaire that has been done for 7 sectors and 10 criteria.	Difference of MOORA method from TOPSIS, normalized performance values are found by subtracting the useless criteria from the total of useful criteria specified according to the type of problem.	The significance of criteria has been determined by using the fuzzy AHP method while the sectors that have been preferred mostly within the certain criteria have been determined using the fuzzy MOORA method. While applying fuzzy AHP, the approach proposed by Chang is also used (Chang, 1996).	Triangle FN is used.	The fuzzy reference point is calculated via max-min approach.	Min-Max Metric of Tchebycheff	The evaluating values should be transformed into performance values that are not fuzzy (Best Non-fuzzy performance/BNP). Calculated values are ranked from the biggest to the smallest and an evaluation is done among the alternatives.
Taylan et al., 2014	The study uses novel analytic tools to evaluate the construction projects and their overall risks under incomplete and uncertain situations. The relative importance index (RII) method was applied to prioritize the project risks based on the data obtained.	Fuzzy AHP (FAHP) was used to create favorable weights for fuzzy linguistic variable of construction projects overall risk. While applying fuzzy AHP, the approach proposed by Chang is also used (Chang, 1996).	The criteria weights are calculated using fuzzy AHP.	Triangular FN is used.	The max-min approach is used for FPIS and FNIS.	The sum of the squared differences is used for the distance.	Alternatives are ranked using fuzzy TOPSIS.
Calık, 2021	This paper aims to develop a new group decision-making approach based on Industry 4.0 components for selecting the best green supplier by integrating AHP and TOPSIS methods under the Pythagorean fuzzy environment.	Using Pythagorean fuzzy sets.	The criterion weights are calculated using fuzzy AHP	Pythagorean fuzzy set is used in the evaluation.	FPIS and FNIS vectors are calculated using the max-min approximation.	Distance calculation is calculated in a vertex method distance-like way according to PFS.	Ranking of alternatives is calculated using fuzzy TOPSIS

Omar et al., 2022	This article integrates the Fuzzy-AHP-TOPSIS system with the quality tool QFD to enable incorporating additional factors and factors' interdependency into the problem model without complicating the solution procedure.	Fuzzy AHP, QFD, and TOPSIS, such that each constituent system overcomes a difficulty as well until reaching the final decision. Fuzzy system keeps escorted to the other three systems to control uncertainties over the solution journey.	Fuzzy AHP-QFD weighting.	The triangular FNs are used	0-1 approach is used for FPIS and FNIS.	The square mean of the differences of the parameters (vertex method) is used as a fuzzy distance.	Final ranking is based on the relative closeness.
Sultana et al., 2015	The paper aims to select the best supplier providing the highest satisfaction for the criteria determined.	Integrating Fuzzy Delphi method with fuzzy AHP and Fuzzy TOPSIS based approach to select the best alternative.	The criterion weights are determined by fuzzy AHP.	The triangular FNs	The max-min approach is used for FPIS and FNIS.	The fuzzy distance is calculated using the vertex method.	Fuzzy TOPSIS. Closeness Coefficient approach
Ramavandi et al., 2021	This paper aims to introduce a novel risk assessment and prioritization model, using hybrid AHP and VIKOR methods in a fuzzy environment.	MCDM is used with aggregation (min, mean, mean, max).	The criteria weights are calculated with FAHP. Then the values are defuzzified and VIKOR is used.	Trapezoidal FN are used	The max-min approach is used for FPIS and FNIS.	Absolute distance on defuzzified values.	Sorting and ranking the alternatives w.r.t. index value using Mean group utility and maximum Regret values.
Liu et al., 2017.	This paper proposes a method for ranking products through online reviews based on sentiment classification and the interval-valued intuitionistic fuzzy TOPSIS.	The method consists of two parts: (1) identifying sentiment orientations of the online reviews based on sentiment classification and (2) ranking alternative products based on interval-valued intuitionistic fuzzy TOPSIS.	Weights of criteria are equal	Evaluation matrix consists of intuitionistic FN values.	max-min approach was used for FPIS and FNIS.	The weighted (w.r.t. criteria) Euclidean distance was used as the fuzzy distance.	Closeness coefficient.
Esmaili-Dooki et al., 2017	The paper presents a real case study to make the best decision of selecting the bank chief inspectors.	The method is a hybrid method of fuzzy AHP and fuzzy TOPSIS to select the best alternative based on some various qualitative and quantitative criteria with different priorities.	The criteria weights are calculated using fuzzy AHP.	The triangular FN are used.	Max-min is used for FPIS and FNIS.	The distance is calculated as the square mean of the parameter differences (the vertex method).	Alternatives are ranked according to their closeness criteria values..

Collan et al., 2015	The presented method is numerically applied to a research and development project selection problem and the effect of using two new closeness coefficients based on multidistance and fuzzy entropy is numerically illustrated.	The new closeness coefficients are based on multidistance or fuzzy entropy, are able to take into consideration the level of similarity between analyzed criteria, and can be used to account for the consistency or homogeneity. The ordered weighted average (OWA) operator is used in the aggregation process over the fuzzy similarity values. A range of orness values is considered in creating a fuzzy overall ranking for each object, after which the fuzzy rankings are ordered to find a final linear ranking.	Aggregation of MCDM weights using (min, mean, mean, max) approach.	Generalized trapezoidal FNs.	max-min approach was used for FPIS and FNIS.	The closeness coefficients based on multidistance or fuzzy entropy are used.	The OWA operator is used in the aggregation process over the fuzzy similarity values. Final ranking is performed among the fuzzy TFN rank values.