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# PARAMETRIC INVESTIGATION OF ALCOHOL CONSUMPTION IN DIABETIC INDIVIDUALS USING THE GEGENBAUER WAVELET METHOD

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**Abstract.** The aim of this study is to introduce a wavelet-based numerical approach to explore the dynamics of alcohol consumption among diabetic individuals and quantify the sensitivity of key model parameters. We consider a six-compartment model of alcohol-induced health complications and analyze the impact of the model parameters on the system using the Gegenbauer wavelet method. This technique is highly efficient and results in accurate solutions without the need for linearization. We validate its precision by calculating residual errors ($\mathcal{O}(10^{-13})$) and comparing the results with established solvers (RK4 and NDSolve). We observe excellent agreement among these methods. Our main contribution is a quantitative analysis of the model parameters using normalized sensitivity coefficients. We justify the model parameters using recent epidemiological data. We observe that social interaction parameters show moderate to high sensitivity, which confirms their important role in driving the progression to harmful drinking. The treatment enrollment parameters show protective effects. Early intervention in the moderate drinking stage shows benefits across multiple compartments, while treatment access for severe complications shows the highest sensitivity coefficient. These findings show that interventions focusing on reducing social interactions and improving treatment accessibility are effective strategies to mitigate alcohol-related harm in diabetic populations.

**Keywords**: Alcohol drinking problem, Diabetic individuals, Gegenbauer wavelets, Parametric investigation
**AMS Subject Classification:** 92B05, 65T60.
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## 1 Introduction

Alcoholism remains a critical area of public health research, responsible for millions of deaths worldwide and representing a significant fraction of the global burden of disease according to World Health Organization (2018, 2019). Although a global threat, the burden of harmful alcohol use is disproportionately borne by vulnerable populations, particularly those managing chronic diseases such as diabetes. For these individuals, the risks are much higher, as alcohol directly interferes with blood sugar regulation, liver function, and the cardiovascular system, which creates serious interactions with diabetic pathophysiology and affects treatment efficacy Vertava Health (2023).

Mathematical modeling has become a powerful tool for simulating the dynamics of alcohol consumption and evaluating intervention strategies Khajji et al. (2020b); Sharma & Samanta (2013); Adu et al. (2017); Agrawal et al. (2018). The existing literature has explored various aspects of alcoholism, including the effects of peer influence Buonomo & Lacitignola (2014), the

impact of treatment programs Khajji et al. (2020a), and the dynamics within specific contexts such as the spread of HIV/AIDS Thomas & Lungu (2010) or the influence of social networks Huo et al. (2019); Huo & Zhang (2016). Some studies have even begun to investigate alcohol use among diabetics, often through the lens of optimal control to manage drug interactions Imken & Fatmi (2024); Essounaini et al. (2023).

Despite these significant advances, a detailed parametric investigation of a model specifically focused on the progression of diabetic complications is a relatively underexplored area. It is not yet fully clear which social or clinical factors are the most critical parameters to prevent the transition from moderate drinking to severe end-stage health outcomes in this high-risk population.

The Gegenbauer wavelet method provides a global and semi-analytical solution in the form of a piecewise polynomial series and yields high accuracy and computational efficiency without the need for linearization Beler et al. (2023); Gümgüm et al. (2019a,b); Özaltun et al. (2022); Özaltun Simşek et al. (2024). We validate our results by comparing them with the ones obtained from the RK4 method and Mathematica's default solver (NDSolve). To demonstrate the performance of our results, we also calculate the residual error for each compartment. Afterwards, we use this method to investigate the impact of the parameters on alcohol-induced diabetes complications. Our aim is to provide quantitative understanding of the key drivers of the epidemic and to identify the most important parameters to prevent the harmful effects of alcohol consumption.

The remainder of this paper is organized as follows. Section 2 presents the biological model, its parameters, and the governing system of equations. Section 3 describes the Gegenbauer wavelet method and its application to the system. In Section 4, we present and analyze the numerical results of our simulations. Finally, Section 5 summarizes our key findings and their implications.

## 2 Mathematical Model

We consider the compartmental model given in Imken & Fatmi (2024) to analyze the effects of the parameters on the dynamics of alcohol consumption and its associated health outcomes within a diabetic population. The total population $N$ is divided into six populations representing different clinical and behavioral states. The population is categorized as follows:

$P(t)$: Potential Drinkers (Diabetic, No Complications). This group consists of individuals with diabetes who do not have advanced complications and do not currently consume alcohol. They are susceptible to initiating alcohol use through social interaction with moderate drinkers (group $M$), transitioning at a rate of $a_1$. A subset of this group may also develop diabetic complications independent of alcohol, moving to group $I$ at a rate of $a_4$.

$M$: Moderate Drinkers (Diabetic, No Complications). This group comprises diabetic individuals who consume alcohol moderately and have not yet developed severe complications. From this state, individuals may either escalate their consumption and transition to the heavy-drinker group ($H$) at a rate of $a_2$, or they may seek help and enter a treatment program ($T$) at a rate of $b_1$.

$I$: Insulin-Treated Non-Drinkers (Diabetic, with Complications). This group includes individuals who, like group $P$, do not consume alcohol but are already managing pre-existing diabetic complications, often requiring insulin treatment. Social interaction with moderate drinkers (group $M$) may precipitate a relapse or initiation of drinking, leading to a rapid progression to severe health issues and a transition to group $D$ at a rate of $a_5$.

$H$: Heavy Drinkers (Diabetic, with Medication). This group represents individuals with an established pattern of heavy alcohol consumption who are concurrently managing their diabetes with anti-diabetic medication. A proportion, $a_3$, of this group will develop severe complications and transition to group $D$. Others may seek professional help, moving to treatment centers ($T$) at a rate of $b_2$. This group also faces an increased mortality risk ($d_1$) from acute events like

hypoglycemia.

$D$: Drinkers with Severe Complications. This group consists of individuals suffering from severe, often irreversible complications such as cardiovascular disease, resulting from the synergistic effects of diabetes and heavy alcohol use. A fraction of this group, $b_3$, may be motivated by their deteriorating health to enter treatment ($T$). This group carries the highest disease-specific mortality rate, $d_2$.

$T$: Individuals in Treatment. This group represents a consolidated state for individuals from groups $M$,$H$, and $D$ who have entered treatment centers to address their alcohol consumption. This state is not without risk, as it includes a mortality rate, $d_3$, accounting for factors such as treatment failure or systemic issues within care facilities.

All compartments are also subject to a natural mortality rate, denoted by $\mu$. Figure 1 presents the interactions and transitions described above:

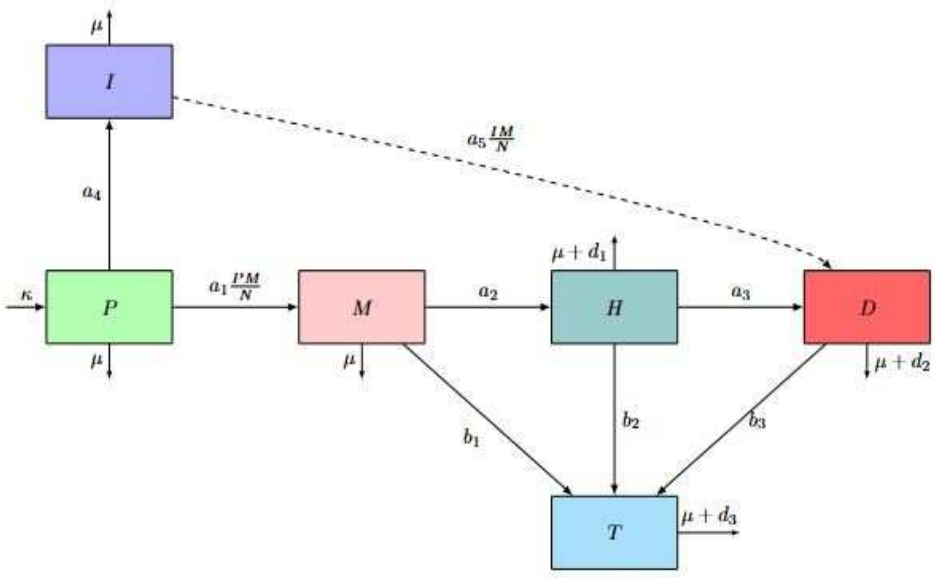


**Figure 1:** A flowchart of the model

We can express this as a system of nonlinear ordinary differential equations as follows:

$$\begin{aligned}
\frac{dP}{dt} &= \kappa - a_1\frac{PM}{N} - (a_4+\mu)P \\
\frac{dM}{dt} &= a_1\frac{PM}{N} - (a_2+\mu+b_1)M \\
\frac{dI}{dt} &= a_4P - a_5\frac{IM}{N} - \mu I \\
\frac{dH}{dt} &= a_2M - (b_2+a_3+\mu+d_1)H \\
\frac{dD}{dt} &= a_3H - (\mu+d_2+b_3)D + a_5\frac{IM}{N} \\
\frac{dT}{dt} &= b_1M + b_2H + b_3D - (\mu+d_3+b_4)T
\end{aligned} \tag{1}$$

with the initial conditions given in Table 1

**Table 1:** Initial Conditions and Sources

| Initial Conditions | | Source |
|---|---|---|
| $P(0)$ | 300 | Represents 31% of population which is consistent with observations that significant proportions of diabetics abstain from alcohol due to health awareness Ahmed et al. (2006). |
| $M(0)$ | 220 | Represents 22% of population, aligned with findings that approximately 50.8% of diabetics consume alcohol Ahmed et al. (2006). |
| $I(0)$ | 200 | Represents 20% of population, reflecting substantial subset of diabetics requiring intensive management who abstain from alcohol Polsky & Akturk (2017). |
| $H(0)$ | 100 | Represents 10% of population, consistent with estimates that 20% of diabetic drinkers engage in heavy drinking National Institute on Alcohol Abuse and Alcoholism (2024). |
| $D(0)$ | 88 | Represents 9% of population, reflecting those who have already progressed to advanced complication stages Kang et al. (2022). |
| $T(0)$ | 70 | Represents 7% of population, aligned with finding that only $7.2-7.9\%$ of people with AUD receive treatment annually National Institute on Alcohol Abuse and Alcoholism (2024). |

**Table 2:** Parameter Justification and Sources

| Parameter | Value | Source |
|---|---|---|
| $\kappa$ | 56 | Recruitment rate balancing natural mortality at steady state Imken & Fatmi (2024). |
| $\mu$ | 0.056 | Natural mortality (approximately 2% annually), adjusted for diabetic populations International Diabetes Federation (2024). |
| $a_1$ | 0.77 | Rate at which potential drinkers become moderate drinkers through social contact. Peer drinking behavior predicts individual consumption Strowger et al. (2024). Calibrated to match observed prevalence. |
| $a_2$ | 0.15 | Rate at which moderate drinkers become heavy drinkers. About $15-20\%$ of moderate drinkers escalate National Institute on Alcohol Abuse and Alcoholism (2024). |
| $a_3$ | 0.10 | Rate at which heavy drinkers develop severe complications. Diabetes and alcohol create synergistic risks Polsky & Akturk (2017). |
| $a_4$ | 0.02 | Rate at which potential drinkers develop complications (moving to group I). Slower progression without alcohol International Diabetes Federation (2024). |
| $a_5$ | 0.22 | Rate at which insulin-treated individuals develop severe complications through contact with moderate drinkers. Pre-existing complications worsen rapidly Polsky & Akturk (2017). |
| $b_1$ | 0.02 | Rate at which moderate drinkers enter treatment. Most do not seek treatment National Institute on Alcohol Abuse and Alcoholism (2024). |
| $b_2$ | 0.009 | Rate at which heavy drinkers enter treatment. Lower than $b_1$ due to treatment barriers Cohen et al. (2007). |
| $b_3$ | 0.03 | Rate at which individuals with severe complications enter treatment. Slightly higher as health deterioration motivates action Cohen et al. (2007). |
| $b_4$ | 0.02 | Rate at which individuals successfully complete treatment and recover. Modest outcomes across programs National Institute on Alcohol Abuse and Alcoholism (2024). |
| $d_1$ | 0.003 | Mortality for heavy drinkers from alcohol-diabetes interactions Polsky & Akturk (2017). |
| $d_2$ | 0.002 | Mortality for severe complications. Lower than $d_1$ due to intensive supervision International Diabetes Federation (2024). |
| $d_3$ | 0.003 | Mortality in treatment centers from withdrawal and treatment failure International Diabetes Federation (2024). |

The nonlinear terms $a_1PM$ and $a_5IM$ in Eq. 1 represent the influence of social interactions between diabetic individuals that drive the progression of alcohol consumption. The model

parameters are defined and justified in Table 2. All parameter values are initially adapted from Imken & Fatmi (2024) and further justified through integration with recent epidemiological literature. Rate parameters are expressed as day$^{-1}$ (per day). The total initial population is $N_0 = 978$ individuals. These parameter values are chosen as a starting point. Our aim is to explore the impact of each parameter on the system's dynamics.

# 3 The Gegenbauer Wavelet Method and its Implementation

In this section, we present the definition of the Gegenbauer wavelets and their applications on the model indicated.

## 3.1 Gegenbauer Polynomials and Wavelets

Gegenbauer wavelets are constructed from an orthonormal basis of functions derived from Gegenbauer polynomials which are orthogonal polynomials.

Gegenbauer polynomials, denoted by $G_m^{(\lambda)}(t)$, are defined in the interval $[-1, 1]$ and are orthogonal with respect to the weight function $(1-t^2)^{\lambda-1/2}$. They represent a generalization of several important polynomial families, including the Legendre and Chebyshev polynomials, and are valued for their excellent convergence properties in approximation theory Szegő (1975).

Gegenbauer wavelets are defined in the interval $[0, 1)$ and are characterized by three indices: a positive integer $l$ for the resolution parameter, $r = 1, 2, \ldots, 2^{l-1}$ for the translation or subinterval, and $m = 0, 1, \ldots, M-1$ for the polynomial degree. The wavelet basis function $\psi_{r,m}(t)$ is formally defined as:

$$\psi_{r,m}(t) = \begin{cases} \frac{1}{\sqrt{L_m^{(\lambda)}}} 2^{(l-1)/2} G_m^{(\lambda)}(2^{l-1}t - r + 1), & \frac{r-1}{2^{l-1}} \leq t < \frac{r}{2^{l-1}}, \\ 0, & \text{otherwise}, \end{cases}$$

where $L_m^{(\lambda)}$ is the normalization constant. The set of Gegenbauer wavelets $\{\psi_{r,m}(t)\}$ forms an orthonormal basis in the Hilbert space $L^2[0, 1)$.

An important property of the wavelet method is its ability to convert the system of differential equations into algebraic matrix operations.

One can approximate a square-integrable function $f(t)$ defined in $[0, 1)$ by a truncated Gegenbauer wavelet series as follows:

$$f(t) \approx \sum_{r=1}^{2^{l-1}} \sum_{m=0}^{M-1} \omega_{rm} \psi_{rm}(t) = \Omega^T \Psi(t), \tag{2}$$

where $\Omega$ is the $2^{l-1}M \times 1$ vector of unknown wavelet coefficients and $\Psi(t)$ is the $2^{l-1}M \times 1$ vector of the Gegenbauer wavelet basis functions.

The derivative of the wavelet basis vector $\Psi(t)$ can be approximated by using operational matrix of differentiation, $A$:

$$\frac{d\Psi(t)}{dt} \approx A\Psi(t). \tag{3}$$

where $A$ is a sparse and well defined matrix Özaltun et al. (2022).

## 3.2 Implementation to the Model

The Gegenbauer wavelet method is applied directly to the system of ODEs in Eq. (1) following the definitions in equations (2) and (3) . Each of the six state variables is represented by its wavelet series expansion, and its derivative is consequently approximated using the operational

matrix $A$:

$$\begin{aligned} P(t) \approx \Omega_1^T \Psi(t) \quad &\Longrightarrow \quad \frac{dP(t)}{dt} \approx \Omega_1^T A \Psi(t), \\ M(t) \approx \Omega_2^T \Psi(t) \quad &\Longrightarrow \quad \frac{dM(t)}{dt} \approx \Omega_2^T A \Psi(t), \\ &= \\ T(t) \approx \Omega_6^T \Psi(t) \quad &\Longrightarrow \quad \frac{dT(t)}{dt} \approx \Omega_6^T A \Psi(t). \end{aligned}$$

Afterwards, we substitute the above approximations into the governing system of ODEs 1. This procedure transforms each differential equation into an algebraic residual equation. For instance, the first equation for the Potential Drinker population becomes:

$$\Omega_1^T A \Psi(t) - \left[ \kappa - a_1 \frac{(\Omega_1^T \Psi(t))(\Omega_2^T \Psi(t))}{N(t)} - (a_4 + \mu)(\Omega_1^T \Psi(t)) \right] = \mathbf{0}, \tag{4}$$

We express the remaining five compartments as in Eq. 4, then insert the standard collocation points $t_j$ into these equations and finally solve the resulting algebraic system for the unknown coefficient vectors $(\Omega_1, \ldots, \Omega_6)$.

Once these coefficient vectors $\Omega_i$ are determined, the continuous, semi-analytical solution for each state variable is constructed using Eq. (2). This provides a representation of the system's dynamics and forms the basis for the parametric investigation and validation presented in the subsequent sections.

# 4 Method Validation

The nonlinear system of ordinary differential equations in Eq. (1) does not have an analytical solution. Thus, we establish the accuracy and reliability of the proposed Gegenbauer wavelet method (GWM) through two complementary approaches: a comprehensive residual error analysis and a direct comparison with established, high-precision numerical solvers.

## 4.1 Residual Error Analysis

The residual error quantifies the agreement of the approximate solution with the original differential equations. The smaller the residual error, the higher the degree of accuracy.

We obtain the residual functions by substituting the approximate solutions denoted by $\tilde{P}(t), \tilde{M}(t), \ldots, \tilde{T}(t)$ back into the governing equations. For example, the residual function for the Potential Drinker compartment is defined as:

$$Res_P(t) = \frac{d\tilde{P}(t)}{dt} - \left[ \kappa - a_1 \frac{\tilde{P}(t)\tilde{M}(t)}{N(t)} - (a_4 + \mu)\tilde{P}(t) \right].$$

Similar residual functions $(Res_M(t), \ldots, Res_T(t))$ are defined for the other five compartments. Figure 2 illustrates the residual error of each population calculated for polynomial approximations of degrees 8 and 12. As expected, increasing the degree of the polynomial results in a decrease in residual error across all population compartments.

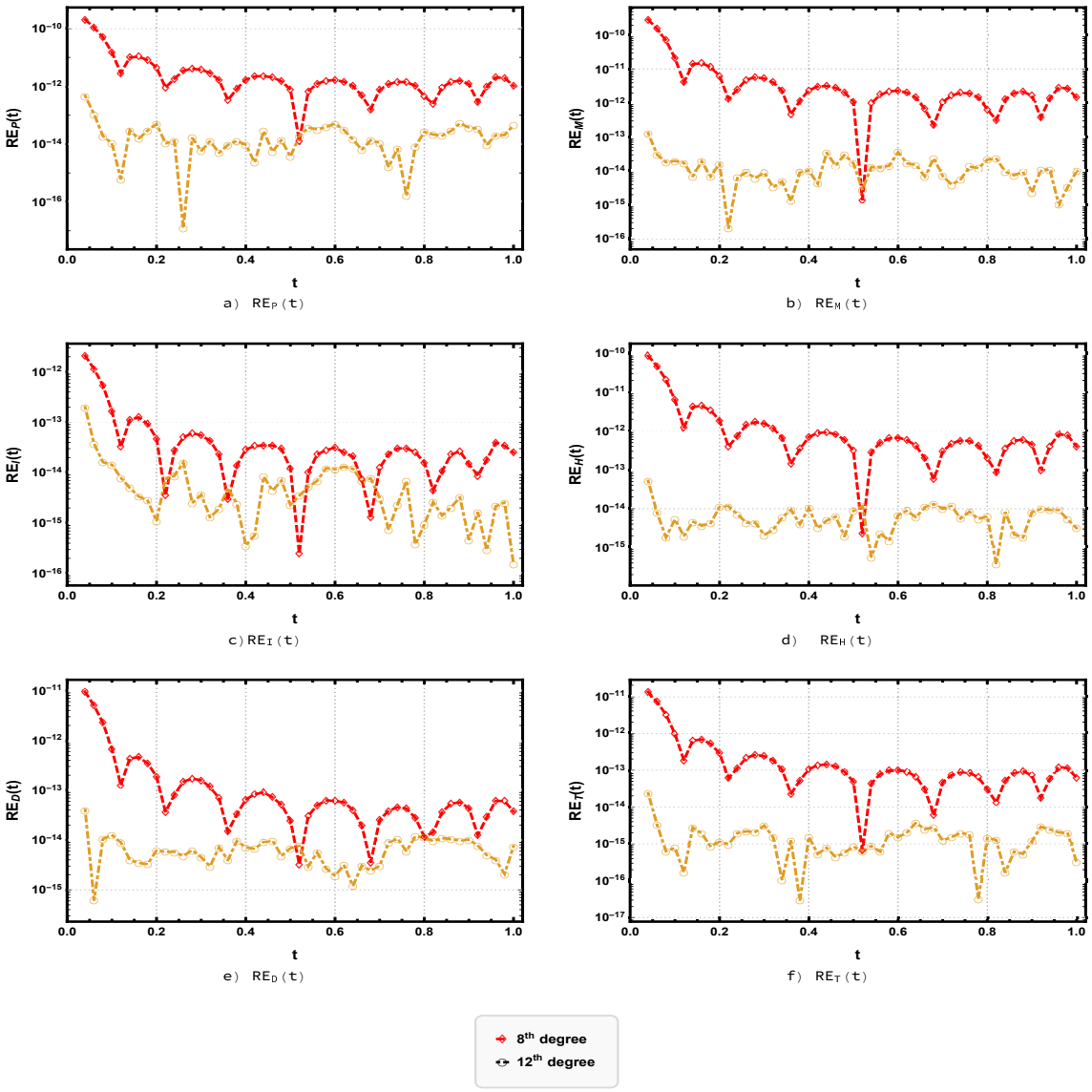


**Figure 2:** Residual error of (a) $P(t)$, (b) $M(t)$, (c) $I(t)$, (d) $H(t)$,(e) $D(t)$, (f) $T(t)$

To quantify the overall error across the entire simulation interval $[0, T]$, we compute the maximum absolute residual error for each compartment:

$$E_{\text{res},X} = \max_{t\in[0,T]} |Res_X(t)|, \quad \text{for } X \in \{P, M, I, H, D, T\}.$$

The maximum residual error for each compartment is presented in Table 3, We observe that maximum residual error for each population is $\mathcal{O}(10^{-13})$ which is very low. This confirms the high accuracy of the GWM in satisfying model equations and illustrates the rapid convergence of the method. Therefore, we may conclude that our proposed method provides high accuracy for the problem indicated.

**Table 3:** Maximum residual error for $M = 13$

| $\max\|RE\|$ | $P(t)$ | $M(t)$ | $I(t)$ |
|---|---|---|---|
| | $4.67743 \times 10^{-13}$ | $1.30403 \times 10^{-13}$ | $1.98659 \times 10^{-13}$ |
| | $H(t)$ | $D(t)$ | $T(t)$ |
| | $5.15001 \times 10^{-14}$ | $4.03437 \times 10^{-14}$ | $2.34917 \times 10^{-14}$ |

### 4.2 Comparison With Other Numerical Methods

In this section, we further validate the accuracy and efficacy of the current method by comparing the results with two numerical techniques, namely the fourth-order Runge-Kutta method (RK4) with a time step of $\Delta t = 0.01$ and the adaptive default solver in Mathematica (NDSolve).

Figure 3 presents the population dynamics for each of the six compartments over an extended period. One can see that all three methods agree very well in all populations, demonstrating the accuracy and reliability of the GWM.

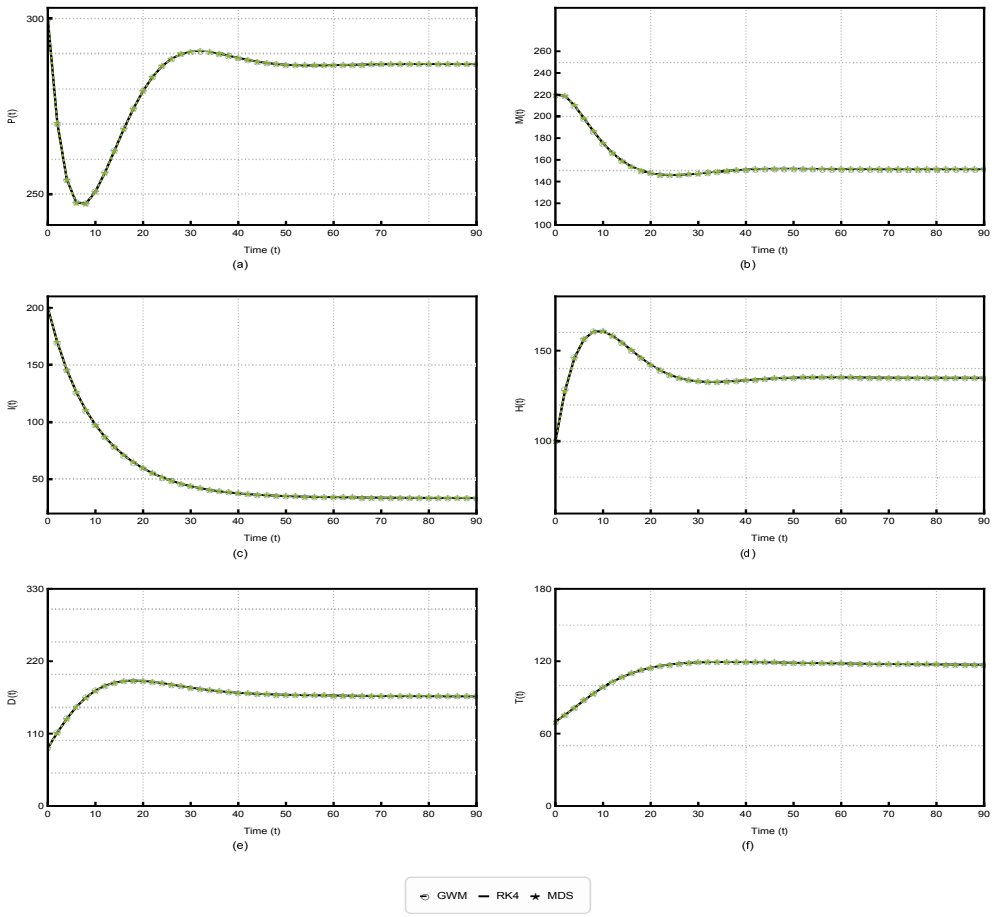


**Figure 3:** Comparison of the GWM, the RK4 method and NDSolve for (a) $P(t)$, (b) $M(t)$, (c) $I(t)$, (d) $H(t)$,(e) $D(t)$, (f) $T(t)$

## 5 Numerical Simulations

In this section, we first analyze and interpret the dynamics of each population presented in Fig. 3, then explore the effect of each parameter on the system.

### 5.1 Population Dynamics

Figure 3(a) represents the dynamics of the Potential Drinker group. This population starts with 300 individuals, then the number decreases over the first ten days due to the transition of individuals to the Moderate Drinker group through social interaction and the development of complications, which moves them to the Insulin-Treated group. Afterwards, the population begins to recover and stabilizes at approximately 285 people, which is lower than its initial state.

Figure 3(b) shows a continuous decline in the number of Moderate Drinkers (M) population from its initial value of 220. This number stabilizes after approximately 50 days at 140 individ-

uals. Although this group receives a constant inflow of new members from the Potential Drinker (P) group, this is not enough to prevent this decrease, which results from the combined rate at which individuals either increase their alcohol consumption to become Heavy Drinkers or seek help at Treatment Centers.

In Figure 3(c), we observe a decrease in the number of Insulin-Treated (I) group, which falls from an initial value of 200 to approximately 30 individuals. The reason for this rapid depletion is the adverse social interaction with Moderate Drinkers. This interaction signifies that the risk of transitioning to the Severe Complications group is proportional to the size of both the Insulin-Treated and Moderate Drinker populations. The simulation demonstrates that the negative influence of group M is substantial, making this population highly vulnerable.

The behavior of the Heavy Drinker group is shown in Figure 3(d). The population initially increases and peaks at approximately 170 individuals within the first 20 days. This is due to the transition from the Moderate Drinker group. However, as the $M$ population decreases, this primary source of inflow decreases as well. Consequently, the H population decreases due to transitioning to $D$, entering treatment centers, or dying. The population eventually stabilizes approximately at 130 individuals.

Figure 3(e) shows that the number of Severe Complications (D) group increases from 88 to 210 during the first 20 days. This initial increase is caused by the transition of Heavy Drinkers who develop complications and the Insulin Treated (I) individuals influenced by social contact. As these H and I populations begin to stabilize, the growth of group D slows and reaches a stable equilibrium where new entries are balanced by outflows to treatment and mortality due to disease complications.

Figure 3(f) presents the dynamics of the populations within the Treatment Centers. We observe an increase from an initial 70 individuals before stabilizing around 125. This growth is a direct result of the combined inflows from the Moderate Drinkers, Heavy Drinkers, and those with Severe Complications. The eventual stabilization does not imply that individuals stop seeking help.

## 5.2 Realistic Dataset Comparison and Parameter Justification

Since a data set for each compartment of the model in Eq. 1 is not available, we ensure that our initial conditions and parameter choices are consistent with the observed prevalence and incidence rates.

### 5.2.1 Initial Conditions and Parameter Justification

We present our initial conditions and parameters in Tables 1 and 2, respectively. Parameter values are initially taken from Imken & Fatmi (2024),then justified using recent epidemiological findings. The total population $N_0 = 978$ allows proportionate distribution across compartments.

We justify our parameter choices using epidemiological observations in the literature. According to Sun et al. (2022), the global diabetes prevalence of approximately 10.5% in adults aged $20 - 79$ support allocating substantial proportions to non drinking diabetic groups. Although approximately 50.8% of diabetics report alcohol consumption Ahmed et al. (2006), our model reflects the lower prevalence often observed in clinical settings where health concerns motivate reduction. The low treatment enrollment rates ($b_1$, $b_2$, $b_3$) align with national data showing only $7 - 8\%$ of people with AUD receive treatment annually National Institute on Alcohol Abuse and Alcoholism (2024).

The social interaction parameters ($a_1$, $a_5$) are adjusted to represent recent findings Strowger et al. (2024). The disease progression parameters ($a_2$, $a_3$) present documented escalation rates and risk of complications from the synergistic effects of diabetes and alcohol use Polsky & Akturk (2017). Some parameters are estimated from model calibration where direct empirical evidence is limited.

#### 5.2.2 Long-term Model Behavior and Epidemiological Consistency

The model predicts a stable endemic state where a significant portion of the population ($P$ and $I$) are non-drinkers, while distinct proportions are moderate, heavy, or suffering severe complications. This distribution agrees with observed patterns where approximately half of diabetic individuals report drinking, with varying levels of consumption Ahmed et al. (2006).

The substantial decline in the Insulin-Treated Non-Drinkers ($I$) group when exposed to moderate drinkers (parameter $a_5$) and the subsequent increase in Drinkers with Severe Complications ($D$) mirrors clinical observations. Pre-existing diabetic complications combined with alcohol use rapidly worsen health outcomes Polsky & Akturk (2017).

The low but persistent population in the Treatment ($T$) group reflects the ongoing need for alcohol treatment services. The model's prediction that treatment seeking remains relatively low compared to the burden of the disorder is consistent with national data showing that 19.3% of people classified as needing substance use treatment received it in 2024 Grant et al. (2015), with even lower rates specifically for AUD.

The model demonstrates the role of social influence in the progression of drinking behavior. This is strongly supported by recent research documenting that peer influences both in person and through social media are robust predictors of alcohol consumption patterns Strowger et al. (2024).

This comparison shows that the model captures realistic public health dynamics of alcohol consumption in diabetic populations. The parameter values and the resulting endemic states are broadly consistent with the observed prevalence rates and transitions in real-world populations.

### 5.3 Parametric Sensitivity Analysis

In this section, we analyze the sensitivity of parameters to identify the influence of each parameter on the system dynamics. The goal is to find out the most important parameter that affects the system. We vary a single parameter at a time while keeping other parameters constant at their baseline values.

#### 5.3.1 Qualitative Parameter Analysis

**Impact of Social Influence Parameters ($a_1$ and $a_5$).** First, we examine the effect of $a_1$, which is the rate of social interaction between Potential Drinkers (P) and Moderate Drinkers (M). As shown in Figure 4, this parameter has a powerful impact on the initial transition to alcohol consumption. As $a_1$ increases from 0.77 to 0.90, the Potential Drinker population (P) decreases dramatically. The number of people in this group decreases from 250 to nearly 20. This indicates that socializing with moderate drinkers has a negative effect on these individuals. The Moderate Drinker (M) population shows a notable increase at higher values of $a_1$. However, this increase is less pronounced than the decrease in P, suggesting that group M acts as a transient state from which individuals either progress to heavier drinking or seek treatment. Higher values of $a_1$ lead to larger endemic populations in the Heavy Drinker (H), Severe Complications (D) and Treatment (T) groups, as the initial surge of new drinkers propagates through the system. The Insulin-Treated group (I) remains largely unaffected, as its dynamics are not directly affected by the P-to-M transition.

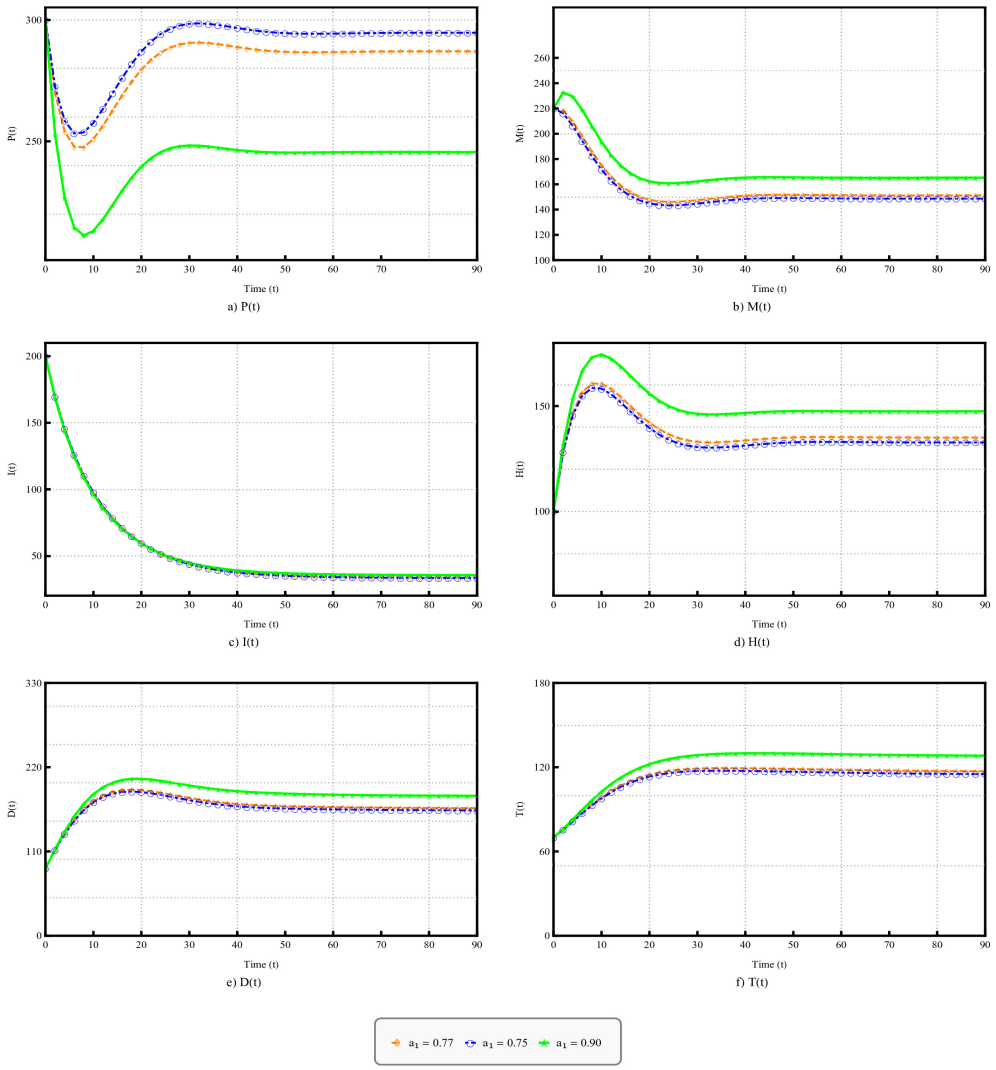


**Figure 4:** Parameter effect of $a_1$ on (a) $P(t)$, (b) $M(t)$, (c) $I(t)$, (d) $H(t)$, (e) $D(t)$, (f) $T(t)$

The parameter $a_5$ represents the contact rate between the highly vulnerable Insulin-Treated group (I) and Moderate Drinkers (M). Figure 5 shows the effect of varying $a_5$ from 0.1 to 0.28. As expected, this parameter directly and profoundly impacts the I and D populations. A higher value of $a_5$ leads to a more rapid and pronounced decline in the I population. Because people in the I group transition to the Severe Complications (D) group rapidly after initiating alcohol use. Consequently, the number of people in the D group increases significantly. Populations that are not directly affected by this interaction (P, M, and H) remain stable. There is a slight increase in the Treatment (T) group at higher values of $a_5$. This indicates that as more individuals enter the D group, a proportion of them are subsequently motivated to seek treatment, highlighting the critical role of intervention even at advanced stages of the disease.

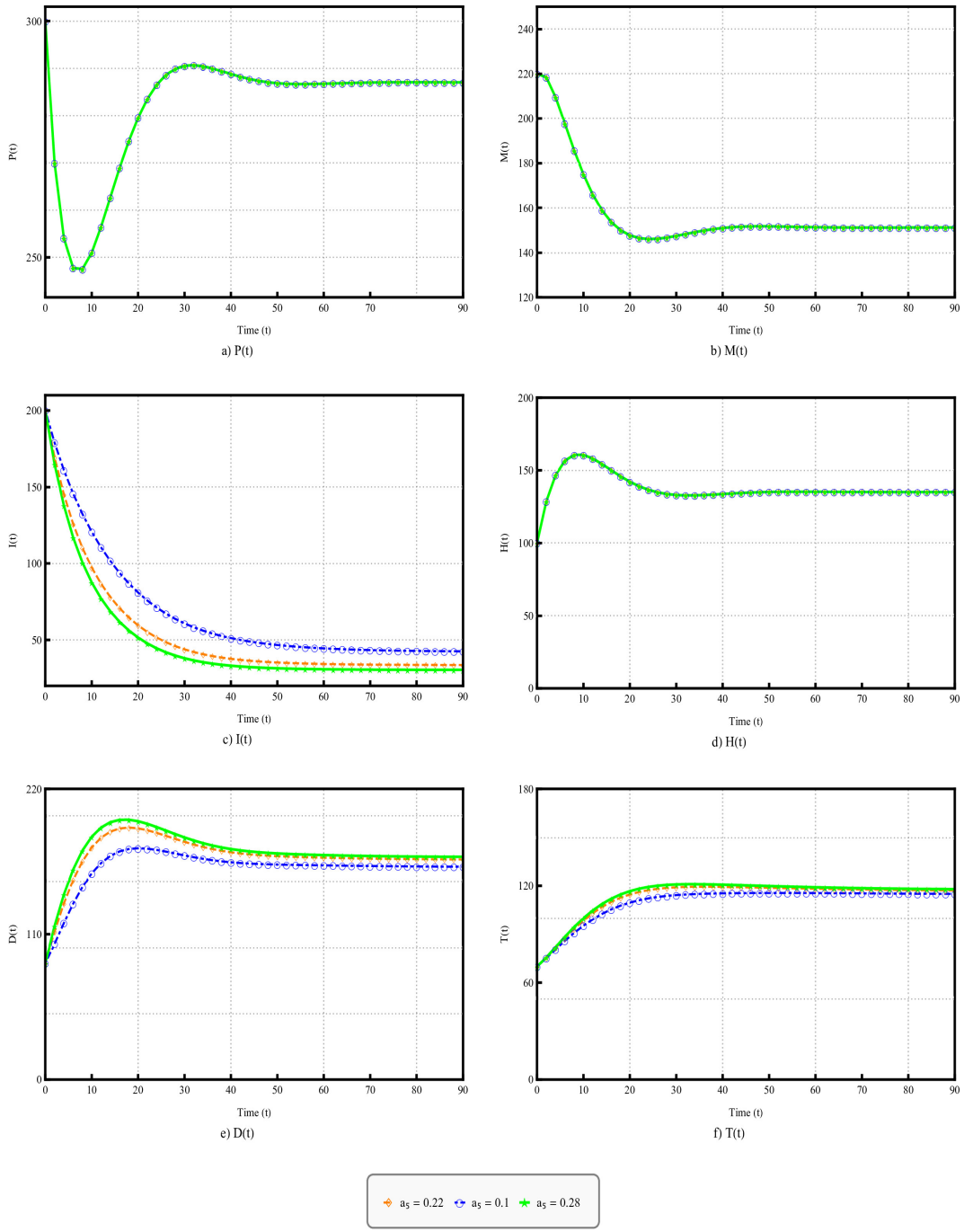


**Figure 5:** Parameter effect of $a_5$ on (a) $P(t)$, (b) $M(t)$, (c) $I(t)$, (d) $H(t)$,(e) $D(t)$, (f) $T(t)$

**Impact of Disease Progression and Treatment Parameters ($a_2$ and $a_3$)** Figure 6 illustrates the response of the system to changes in $a_2$, which represents the rate at which Moderate Drinkers (M) become Heavy Drinkers (H). As $a_2$ increases from 0.12 to 0.155, the Moderate Drinker population (M) decreases significantly. The decrease in the number of the M group directly increases the number of people in the Heavy Drinker (H) group. The number of people in the Potential Drinker (P) group increases as $a_2$ increases. This occurs because the number of people in the Moderate Drinkers group that influence group P decreases more rapidly, thus reducing the overall rate of new entries into the M group and allowing the P group to grow. The populations with complications (I and D) remain largely stable, as they are not directly affected by the M-to-H transition. However, there is a slight decrease in the Treatment (T) population as fewer individuals seek help from the shrinking M group.

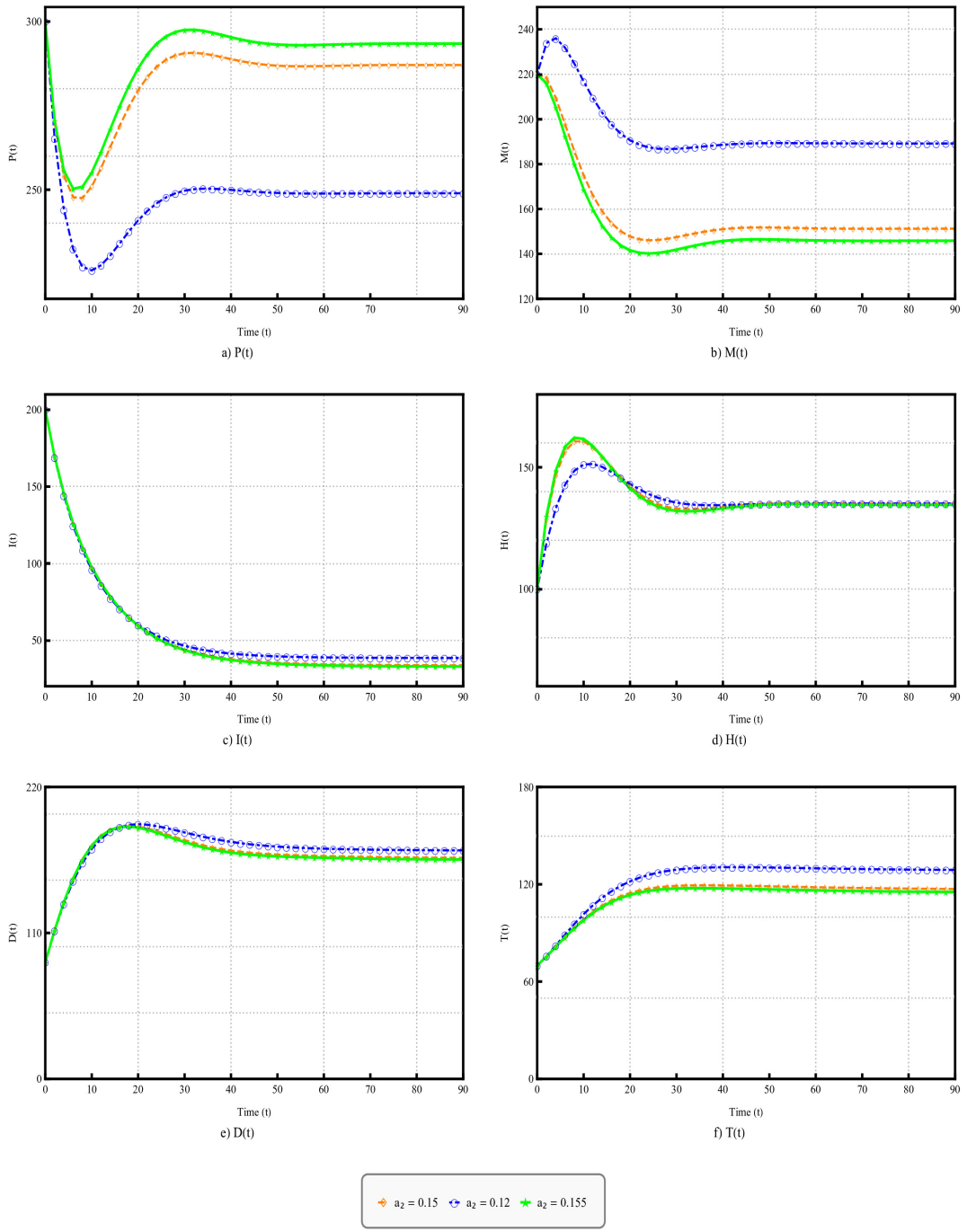


**Figure 6:** Parameter effect of $a_2$ on (a) $P(t)$, (b) $M(t)$, (c) $I(t)$, (d) $H(t)$,(e) $D(t)$, (f) $T(t)$

Figure 7 shows that changing the value of $a_3$ has a negligible effect on non-drinking ($P$, $I$) and moderate drinking ($M$) populations. However, $a_3$ is a critical parameter for the dynamics of the Heavy Drinker ($H$) and Severe Complications ($D$) groups. As expected, a higher value of $a_3$ increases the transition of the Heavy Drinker group into the Severe Complications group. This results in a faster decrease in the number of the H group and correspondingly a significant increase in the number of the D group. In the mean time, the Treatment group (T) experiences an increase. This is because the larger $D$ population, combined with the remaining $H$ population, contributes more individuals to treatment centers.

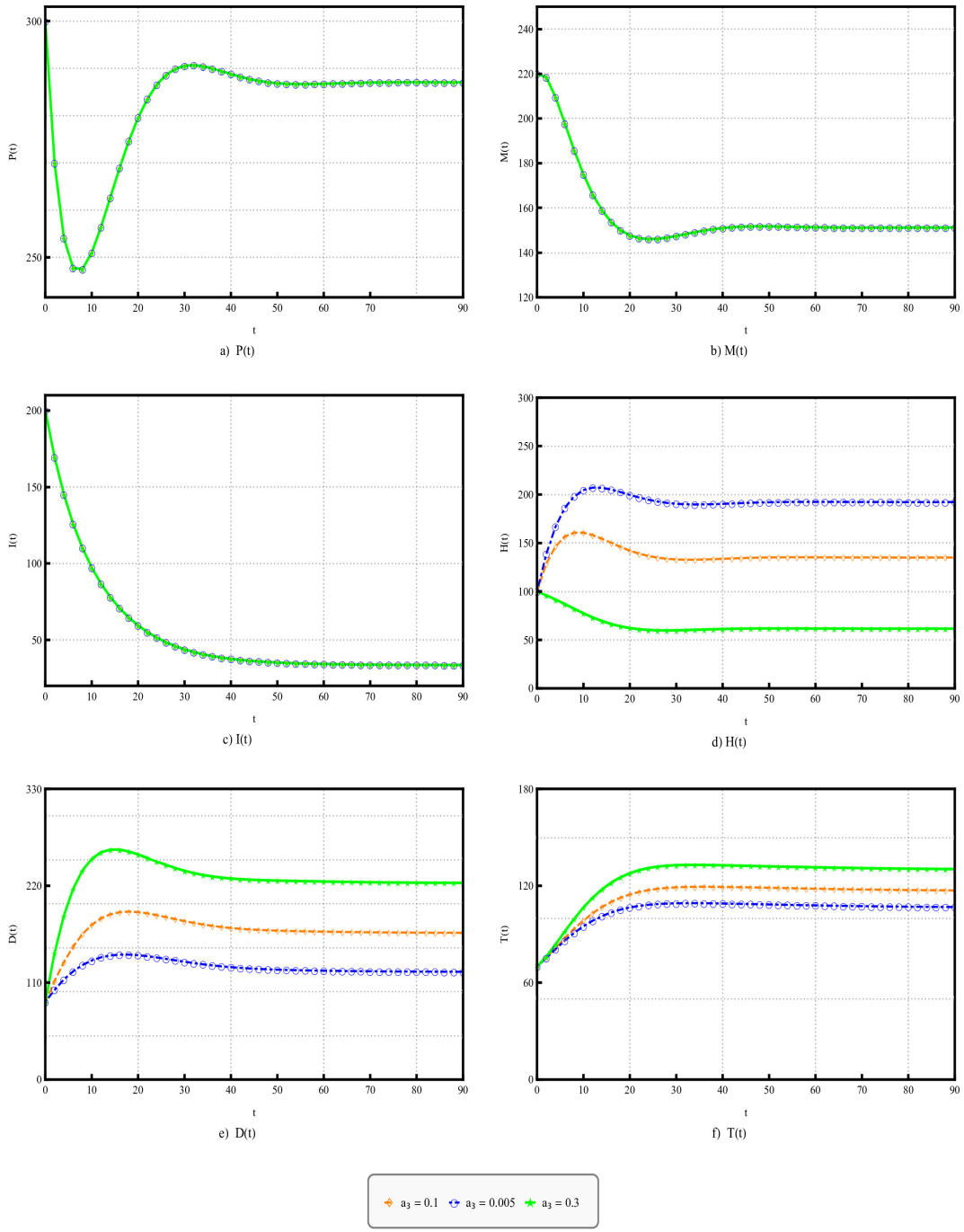


**Figure 7:** Parameter effect of $a_3$ on (a) $P(t)$, (b) $M(t)$, (c) $I(t)$, (d) $H(t)$,(e) $D(t)$, (f) $T(t)$

**Impact of Treatment Enrollment ($b_1$-$b_4$)** Figure 8 illustrates the response of the system to an increase in the rate at which Moderate Drinkers enroll in treatment centers ($b_1$). As $b_1$ increases from 0.01 to 0.025, the Moderate Drinker ($M$) population decreases. This decrease has several positive effects. The number of the $H$ and $D$ groups decreases as fewer individuals transition from the $M$ group into these higher-risk states. The decrease in the number of the $M$ group decreases the social pressure on Potential Drinkers ($P$), allowing the $P$ population to increase. This implies that early intervention at the moderate drinking stage is highly effective, not only for people seeking treatment but also for its broader preventative impact on the entire system.

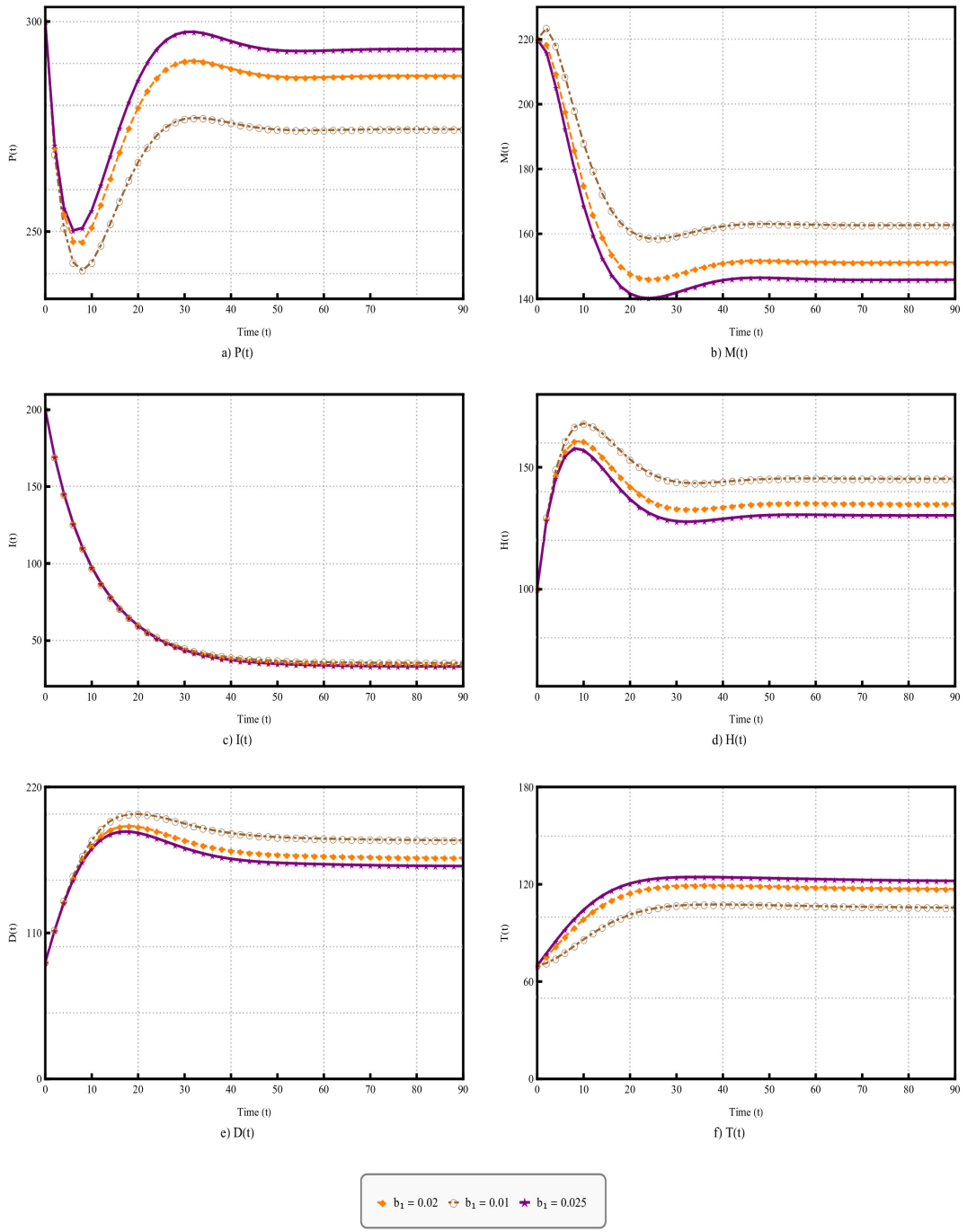


**Figure 8:** Parameter effect of $b_1$ on (a) $P(t)$, (b) $M(t)$, (c) $I(t)$, (d) $H(t)$,(e) $D(t)$, (f) $T(t)$

Figure 9 shows that increasing the rate at which Heavy Drinkers ($H$) enroll in treatment centers ($b_2$) has a direct and beneficial impact on the populations with the highest risk. A higher value of $b_2$ leads to a decrease in the number of the Heavy Drinker (H) and Severe Complications (D) groups. The Treatment (T) population increases as a consequence. It has a negligible effect on the $P$, $M$ and $I$ groups. This finding emphasizes the importance of targeted outreach and support for heavy drinkers.

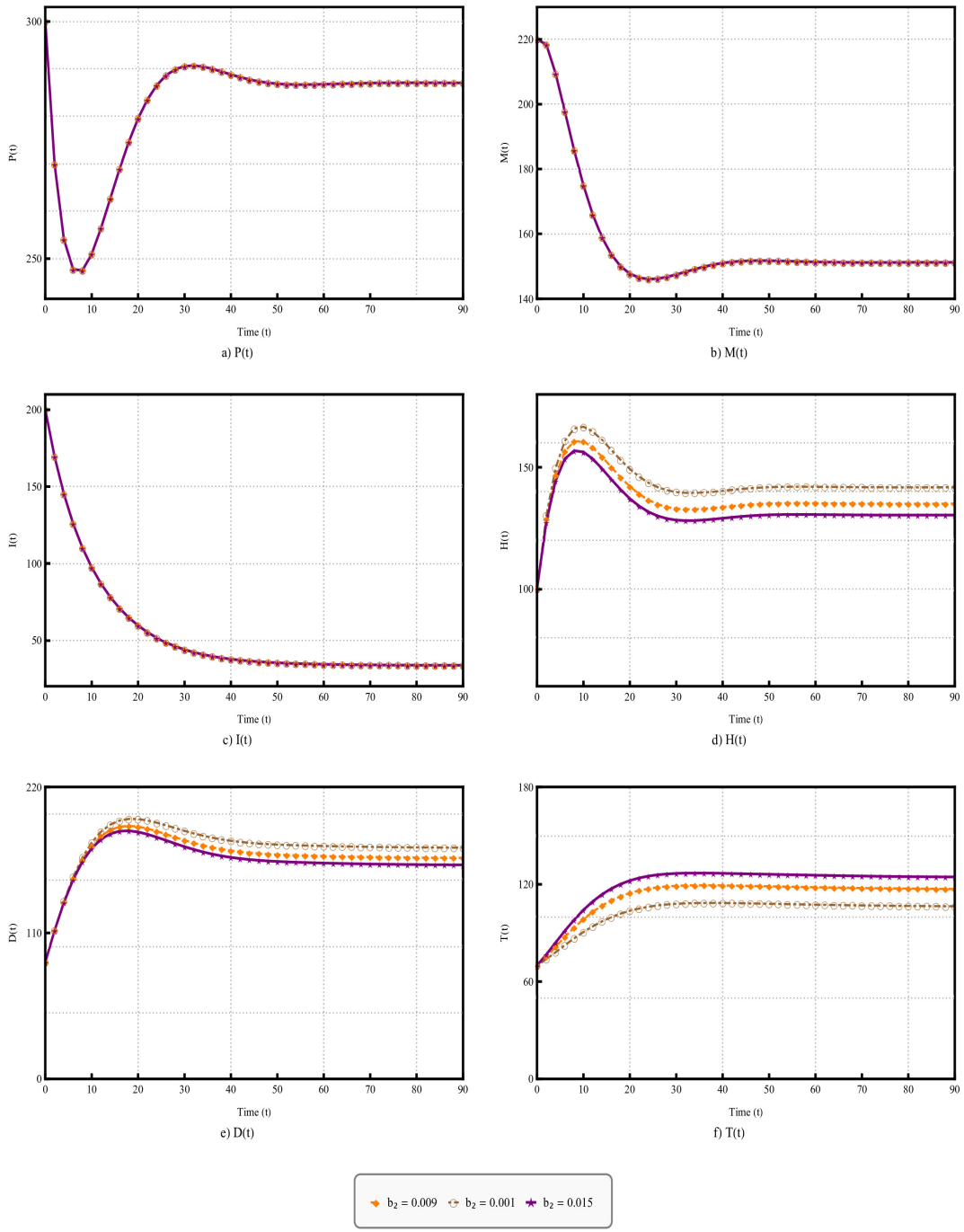


**Figure 9:** Parameter effect of $b_2$ on (a) $P(t)$, (b) $M(t)$, (c) $I(t)$, (d) $H(t)$,(e) $D(t)$, (f) $T(t)$

Figure 10 presents the effect of increasing the treatment enrollment rate ($b_3$) of people who already suffer from Severe Complications ($D$). As expected, a higher value of $b_3$ results in a lower number of the $D$ group and a higher number of the $T$ group. Since the $D$ group does not have a social influence on other groups in this model, changes in this parameter do not affect the $P$, $M$, $I$, or $H$ populations. The benefit is entirely localized to the D group, showing that access to treatment remains a vital component of care even in advanced stages of complication.

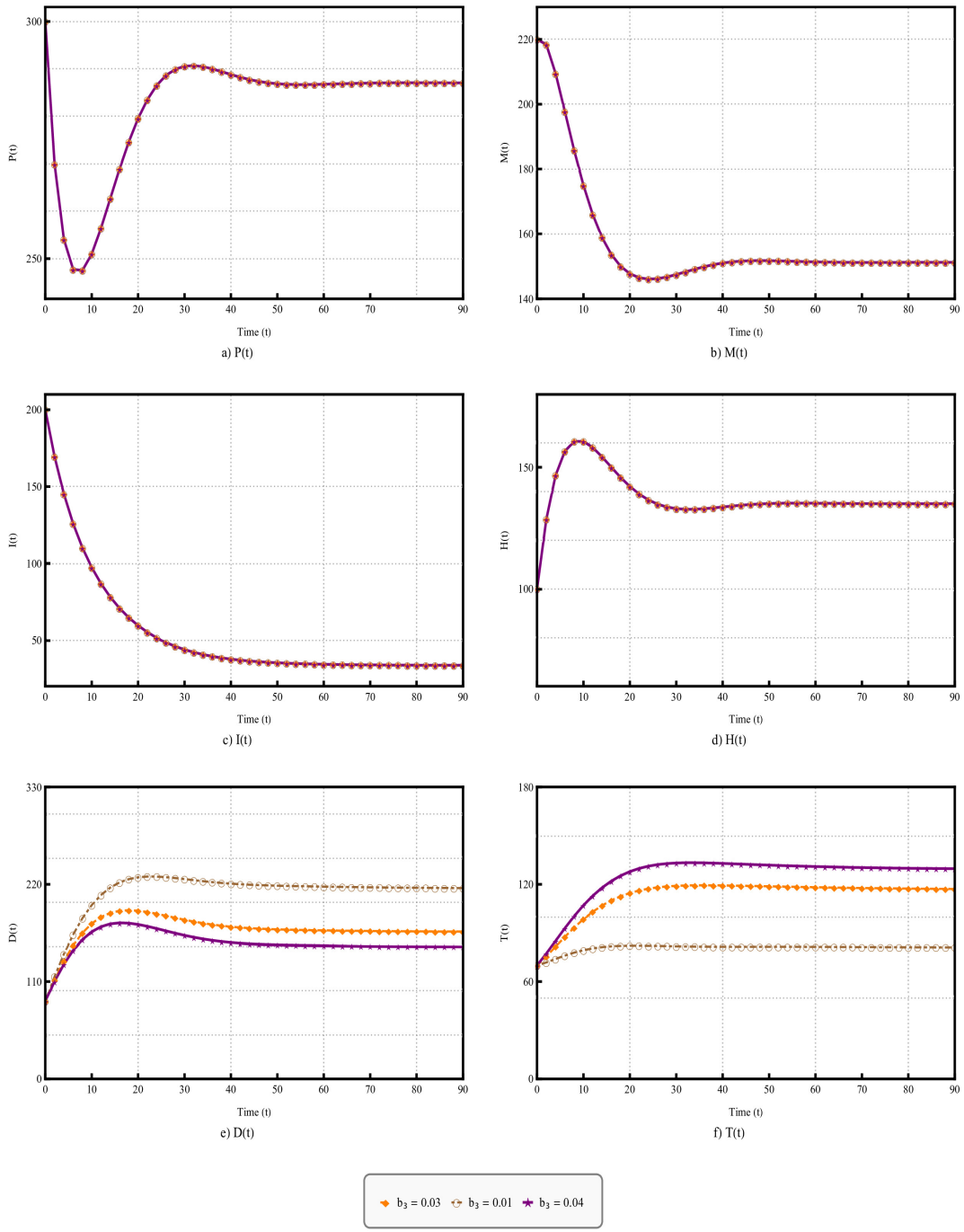


**Figure 10:** Parameter effect of $b_3$ on (a) $P(t)$, (b) $M(t)$, (c) $I(t)$, (d) $H(t)$,(e) $D(t)$, (f) $T(t)$

The parameter $b_4$, which is the rate at which individuals successfully quit alcohol and leave the Treatment ($T$) group, is an ultimate measure of a treatment program's success and its ability to facilitate long-term recovery. Figure 11 shows that increasing the value of $b_4$ leads to a significant decrease in the number of the $T$ group. This reduction represents a higher throughput of the treatment system, where individuals enter, recover then leave the system for good. This result shows that the efficacy of the treatment itself is as critical as enrollment. Investing in evidence-based programs that maximize the success rate ($b_4$) is essential to achieve long-term recovery and reduce the overall public health burden of alcohol use disorder in diabetic populations.

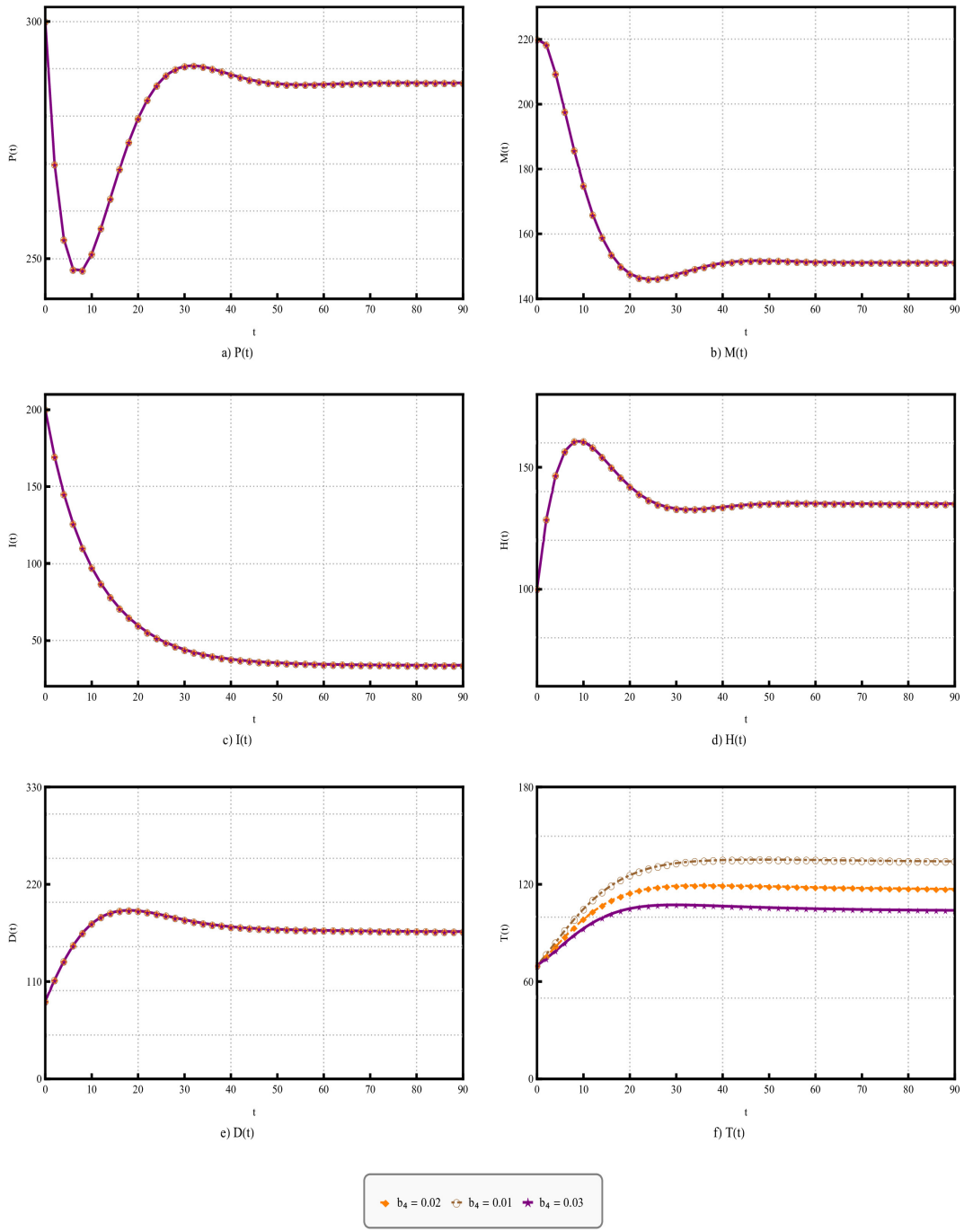


**Figure 11:** Parameter effect of $b_4$ on (a) $P(t)$, (b) $M(t)$, (c) $I(t)$, (d) $H(t)$,(e) $D(t)$, (f) $T(t)$

### 5.3.2 Quantitative Sensitivity Analysis

In this section, we quantify the parameter sensitivity using normalized sensitivity coefficients. The normalized sensitivity coefficient is defined as:

$$S_{X,p} = \frac{\partial X_{ss}}{\partial p} \cdot \frac{p}{X_{ss}} \approx \frac{\%\Delta X}{\%\Delta p} \tag{5}$$

where $X_{ss}$ denotes the steady state value of compartment $X$ and $p$ is the parameter. The magnitude indicates the sensitivity level. $|S_{X,p}| < 0.1$ represent low, $0.1 < |S_{X,p}| < 1$ represents moderate, and $|S_{X,p}| > 1$ represents high sensitivity. The sign indicates the direction of the relationship.

Table 4 presents the normalized sensitivity coefficients for all key parameters across the six compartments. The values are calculated by Eq. 5.

**Table 4:** Normalized Sensitivity Coefficients at Steady State

| **Parameter** | $S_{P,p}$ | $S_{M,p}$ | $S_{I,p}$ | $S_{H,p}$ | $S_{D,p}$ | $S_{T,p}$ |
|---|---|---|---|---|---|---|
| $a_1$ | $-0.747$ | $+0.612$ | 0.000 | $+0.692$ | $+0.690$ | $+0.664$ |
| $a_2$ | $+0.411$ | $-0.866$ | $-0.741$ | $+0.076$ | $-0.476$ | $-0.219$ |
| $a_3$ | 0.000 | 0.000 | 0.000 | $-0.012$ | $+0.014$ | $+0.003$ |
| $a_5$ | 0.000 | 0.000 | $-0.185$ | 0.000 | $+0.074$ | $+0.048$ |
| $b_1$ | $+0.064$ | $-0.063$ | 0.000 | $-0.092$ | $-0.111$ | $+0.091$ |
| $b_2$ | 0.000 | 0.000 | 0.000 | $-0.007$ | $-0.006$ | $+0.010$ |
| $b_3$ | 0.000 | 0.000 | 0.000 | 0.000 | $-0.121$ | $+0.244$ |
| $b_4$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | $-0.093$ |

We observe that the social interaction parameter $a_1$ shows moderate to high sensitivity along five compartments, with coefficients ranging from 0.61 to 0.75 in magnitude. This confirms that social contact between potential and moderate drinkers is a key driver of epidemic progression. The parameter $a_5$ shows moderate sensitivity for the insulin-treated group ($S_{I,a_5} = -0.185$), which means that this population is vulnerable to social influences. The parameter $a_2$ shows the highest magnitude of sensitivity ($S_{M,a_2} = -0.866$) for the moderate drinker compartment. The parameter $a_3$ shows very low normalized sensitivity despite large absolute effects. This is due to the extreme parameter variation range (5900%), which indicates that although absolute changes are substantial, the relative sensitivity is low.

The treatment enrollment parameters show different levels of impact. The parameter $b_1$ shows moderate protective effects across multiple compartments, with sensitivity magnitudes ranging from 0.06 to 0.11. This indicates that early intervention at the moderate drinking stage shows benefits throughout the system. The parameter $b_3$ shows the highest single sensitivity coefficient ($S_{T,b_3} = +0.244$), which confirms that access to treatment centers for severe complications has the strongest relative impact on the treatment population. The parameter $b_2$ shows low sensitivity ($|S| < 0.01$) despite large variations. The parameter $b_4$ affects only the treatment compartment ($S_{T,b_4} = -0.093$), as expected from the structure of the model.

Based on these quantified results, we can list the most influential parameters for public health intervention as $a_1$, $b_1$, and $b_3$, respectively.

## 6 Conclusion

In this study, we introduce a wavelet-based numerical framework to investigate alcohol consumption behaviors among diabetic individuals. We apply the Gegenbauer wavelet method to a six-compartment model of alcohol-induced health complications and analyze the impact of the model parameters on the system. We validate the numerical results by comparing them with other numerical results and observe an excellent agreement among these methods. We also calculate the residual errors of each population to ensure that the proposed method captures the long-term behavior of the epidemic. The residual errors are $\mathcal{O}(10^{-13})$. As a result, we may say that the Gegenbauer wavelet method is a consistent and reliable tool for modeling complex long-term dynamics of diabetic drinker populations.

Our main contribution is the quantitative analysis of the model parameters, which allows us to identify the most important parameters that influence the system. We calculate normalized sensitivity coefficients to quantify the impact of each parameter on the system. We disclose that the rate of influence of moderate drinkers on potential drinkers is a key parameter of the initial surge in alcohol consumption, while the influence on insulin-treated individuals significantly

increases the transition to severe complications. We also observe that the social interaction parameter $a_1$ shows moderate to high sensitivity along five compartments, with coefficients ranging from 0.61 to 0.75 in magnitude. Hence, we may say that social interactions are important drivers of the epidemic.

Our analysis reveals that interventions that increase recruitment into treatment centers create benefits across the system. Early intervention in the moderate drinking stage shows protective effects for multiple compartments (sensitivity magnitudes $0.06 - 0.11$), while access to treatment centers for severe complications shows the highest single sensitivity coefficient (0.244). The treatment success rate is an important measure of a program's ability to facilitate long-term recovery and reduce the public health burden.

The results suggest that interventions focusing on reducing social interactions which promote drinking and improving public awareness are important for mitigating the early progression to heavy drinking. Furthermore, we observe that efforts directed at populations already experiencing advanced complications can still yield benefits, which highlights the need for intervention at multiple stages.

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## References

Adu, I.K., Osman, M. & Yang, C. (2017). Mathematical Model of Drinking Epidemic. *BJMCS*, *22*(5), 1-10. https://doi.org/10.9734/BJMCS/2017/33659

Agrawal, A., Tenguria, A. & Modi, G. (2018). Role of Epidemic Model to Control Drinking Problem. *IJSRMSS*, *5*(4), 341-354. https://doi.org/10.26438/ijsrmss/v5i4.324337

Ahmed, A.T., Karter, A.J., & Liu, J. (2006). Alcohol consumption is inversely associated with adherence to diabetes self-care behaviours. *Diabetic Medicine*, *23*, 795–802. https://doi.org/10.1111/j.1464-5491.2006.01878.x

Beler, A., Özaltun Simşek, G. & Gümgüm, S. (2023). Numerical Solutions of the HIV Infection Model of $CD4^(+)$ Cells by Laguerre Wavelets. *Mathematics and Computers in Simulation*, *209*, 205–219. https://doi.org/10.1016/j.matcom.2023.02.016.

Buonomo, B. & Lacitignola, D. (2014). Modeling peer influence effects on the spread of high–risk alcohol consumption behavior. *Ricerche mat*, *63*, 101–117. https://doi.org/10.1007/s11587-013-0167-3

Cohen, E., Feinn, R., Arias, A. & Kranzler, H.R. (2007). Alcohol treatment utilization: Findings from the National Epidemiologic Survey on Alcohol and Related Conditions. *Drug and Alcohol Dependence, 86*(2-3), 214-221. https://doi.org/10.1016/j.drugalcdep.2006.06.008

Essounaini, A., Khajji, B., Hilal, S., Laarabi, H. & Rachik, M. (2023). Modeling mathematical and analysis of an alcohol drinking with n complications. *Commun Math Biol Neurosci*, *2023*, 57. https://doi.org/10.28919/cmbn/7918

Grant, B.F., Goldstein, R.B., Saha, T.D., Chou, S.P., Jung, J., Zhang, H., ... & Hasin, D.S. (2015). Epidemiology of DSM-5 Alcohol Use Disorder: Results From the National Epidemiologic Survey on Alcohol and Related Conditions III. JAMA Psychiatry, 72(8), 757-766. https://doi.org/10.1001/jamapsychiatry.2015.0584

Gümgüm, S., Özdek, D.E., Özaltun, G. & Bildik, N. (2019a). Legendre wavelet solution of neutral differential equations with proportional delays. *Journal of Applied Mathematics and Computing*, *1*(1-2), 389–404. https://doi.org/10.1007/s12190-019-01256-z.

Gümgüm, S., Özdek, D.E. & Özaltun, G. (2019b). Legendre wavelet solution of high order nonlinear ordinary delay differential equations. *Turkish Journal of Mathematics*, *43*(3), 1339–1352. https://doi.org/10.3906/mat-1901-109.

Huo, H.F., Jing, S.L., Wang, X.Y. & Xiang, H. (2019). Modelling and analysis of an alcoholism model with treatment and effect of Twitter. *Math Biosci Eng*, *16*(5), 3595–3622. https://doi.org/10.3934/mbe.2019179

Huo, H.F. & Zhang, X.M. (2016). Complex dynamics in an alcoholism model with the impact of Twitter. *Math Biosci*, *281*, 24–35. https://doi.org/10.1016/j.mbs.2016.08.009

Imken, I. & Fatmi, N.I. (2024). A new mathematical model of drinking alcohol among diabetes population taking anti-diabetic drugs: an optimal control approach. *Commun Math Biol Neurosci*, *2024*, 5. https://doi.org/10.28919/cmbn/8359

International Diabetes Federation (IDF) (2024). *IDF Diabetes Atlas, 11th edition.* Brussels, Belgium: International Diabetes Federation. Available at: https://diabetesatlas.org/

Kang, S.H., Do, J.Y., Kim, J.C. & Park, J.W. (2022). Association among current smoking, alcohol consumption, regular exercise, and lower extremity amputation in patients with diabetic foot: Nationwide population-based study. *Endocrinology and Metabolism*, *37*(5), 770-780. https://doi.org/10.3803/EnM.2022.1519

Khajji, B., Labzai, A., Balatif, O. & Rachik, M. (2020a). Mathematical Modeling and Analysis of an Alcohol Drinking Model with the Influence of Alcohol Treatment Centers. *Int J Math Math Sci.*

Khajji, B., Labzai, A., Kouidere, A., Balatif, O. & Rachik, M. (2020b). A Discrete Mathematical Modeling of the Influence of Alcohol Treatment Centers on the Drinking Dynamics Using Optimal Control. *J Appl Math*, *2020*, Article ID 9284698, 13 pages. https://doi.org/10.1155/2020/9284698

National Institute on Alcohol Abuse and Alcoholism (NIAAA) (2024). *Alcohol Treatment in the United States.* Available at: https://www.niaaa.nih.gov/alcohols-effects-health/alcohol-topics-z/alcohol-facts-and-statistics/alcohol-treatment-united-states

Özaltun, G., Konuralp, A. & Gümgüm, S. (2022). Gegenbauer wavelet solutions of fractional integro-differential equations. *Journal of Computational and Applied Mathematics*, *420*(09), 114830. https://doi.org/10.1016/j.cam.2022.114830

Özaltun Simşek, G., Beler, A. & Gümgüm, S. (2024). Gegenbauer Wavelet Solutions of the SIR and the SITR Systems of the COVID-19 Disease. *International Journal of Biomathematics.* https://doi.org/10.1142/S1793524524501432

Polsky, S. & Akturk, H.K. (2017). Alcohol consumption, diabetes risk, and cardiovascular disease within diabetes. *Current Diabetes Reports*, *17*(12), 136. https://doi.org/10.1007/s11892-017-0950-8

Sharma, S. & Samanta, G.P. (2013). Drinking as an Epidemic: A Mathematical Model with Dynamic Behaviour. *J Appl Math Informatics*, *31*(1-2), 1-25. https://doi.org/10.14317/jami.2013.001

Strowger, M., Meisel, M.K., Haikalis, M., Rogers, M.L. & Barnett, N.P. (2024). Associations between frequency of exposure to peer-generated alcohol-related posts and alcohol use within a social network of college students. *Addictive Behaviors*, *152*, 107956. https://doi.org/10.1016/j.addbeh.2024.107956

Sun, H., Saeedi, P., Karuranga, S., et al. (2022). IDF Diabetes Atlas: Global, regional and country-level diabetes prevalence estimates for 2021 and projections for 2045. *Diabetes Research and Clinical Practice*, *183*, 109119. https://doi.org/10.1016/j.diabres.2021.109119

Szegő, G. (1975). *Orthogonal Polynomials*, 4th edn. American Mathematical Society Colloquium Publications, Vol. 23. AMS, Providence, RI. See Chapter IV, Section 4.7 for the Gegenbauer polynomials as special cases of Jacobi polynomials.

Thomas, G. & Lungu, E.M. (2010). A two-sex model for the influence of heavy alcohol consumption on the spread of HIV/AIDS. *Math Biosci Eng*, *7*(4), 871-904. https://doi.org/10.3934/mbe.2010.7.871

Vertava Health (2023). Alcohol and diabetes. https://vertavahealth.com/alcohol/diabetes/

World Health Organization (2018). Global status report on alcohol and health. https://ncdalliance.org/resources/who-global-status-report-on-alcohol-and-health

World Health Organization (2019). Global status report on alcohol and health and treatment of substance use disorders. https://www.who.int/publications/i/item/9789240096745